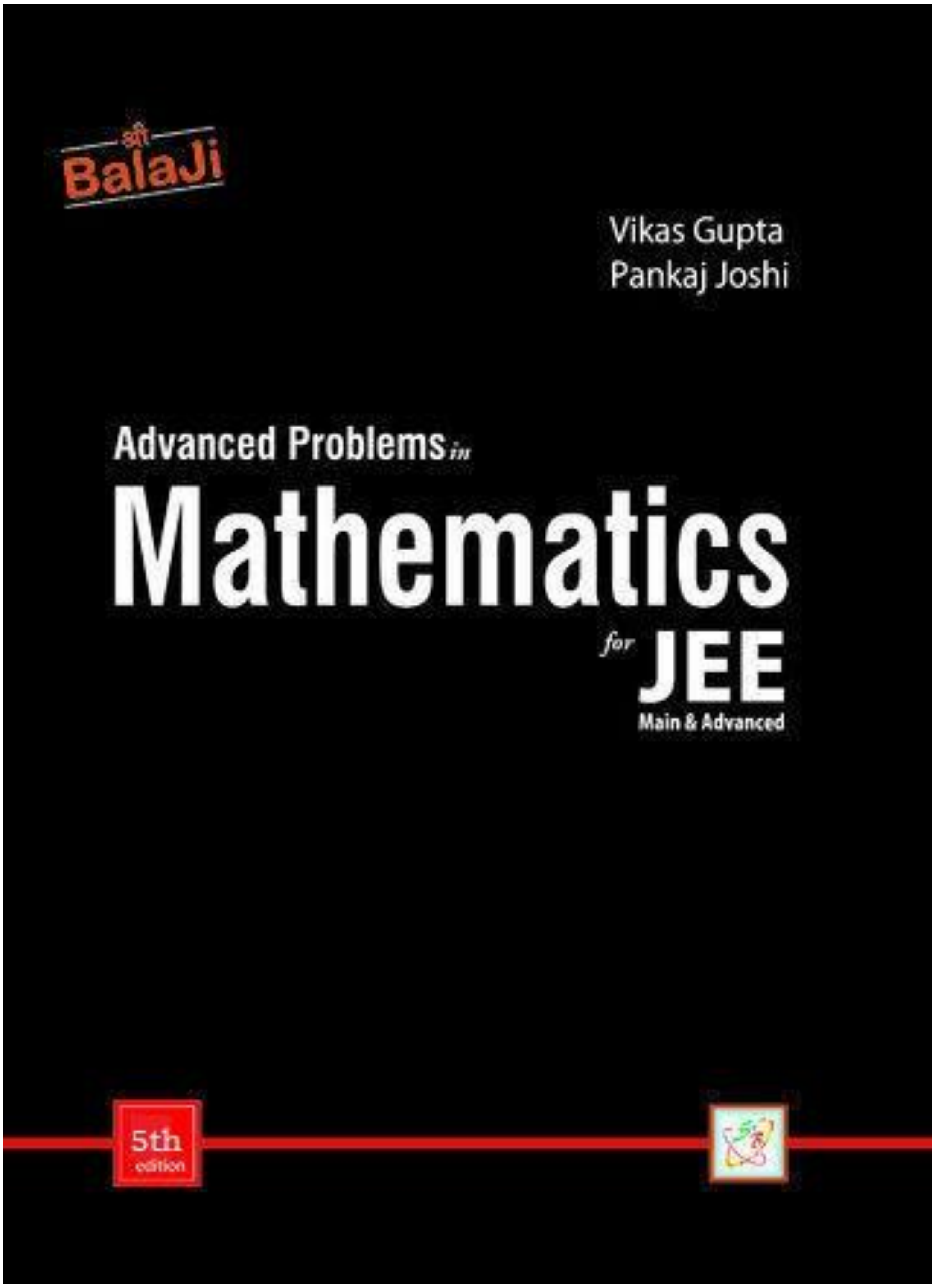
Balaji

Advanced Problems in Mathematics Chapter 1 to 9

for IIT JEE Main and Advanced

by

Vikas Gupta and Pankaj Joshi



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Advanced Problems *in*

MATHEMATICS

for

JEE (MAIN & ADVANCED)

by :

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Chapter 1 – Function

1 FUNCTION

Exercise-1 : Single Choice Problems

1. Range of the function $f(x) = \log_2(2 - \log_{\sqrt{2}}(16\sin^2 x + 1))$ is :

(a) $[0, 1]$ (b) $(-\infty, 1]$ (c) $[-1, 1]$ (d) $(-\infty, \infty)$

2. The value of a and b for which $|e^{|x-b|} - a| = 2$, has four distinct solutions, are :

(a) $a \in (-3, \infty), b = 0$ (b) $a \in (2, \infty), b = 0$ (c) $a \in (3, \infty), b \in R$ (d) $a \in (2, \infty), b = a$

3. The range of the function :

$$f(x) = \tan^{-1} x + \frac{1}{2}\sin^{-1} x$$

(a) $(-\pi/2, \pi/2)$ (b) $[-\pi/2, \pi/2] - \{0\}$ (c) $[-\pi/2, \pi/2]$ (d) $(-3\pi/4, 3\pi/4)$

4. Find the number of real ordered pair(s) (x, y) for which :

$16^{x^2+y} + 16^{x+y^2} = 1$

(a) 0 (b) 1 (c) 2 (d) 3

5. The complete range of values of 'a' such that $\left(\frac{1}{2}\right)^{|x|} = x^2 - a$ is satisfied for maximum number of values of x is :

(a) $(-\infty, -1)$ (b) $(-\infty, \infty)$ (c) $(-1, 1)$ (d) $(-1, \infty)$

6. For a real number x, let $[x]$ denotes the greatest integer less than or equal to x. Let $f : R \to R$ be defined by $f(x) = 2x + [x] + \sin x \cos x$. Then f is :

(a) One-one but not onto (b) Onto but not one-one

(c) Both one-one and onto (d) Neither one-one nor onto

7. The maximum value of $\sec^{-1}\left(\frac{7 - 5(x^2 + 3)}{2(x^2 + 2)}\right)$ is :

(a) $\frac{5\pi}{6}$ (b) $\frac{5\pi}{12}$ (c) $\frac{7\pi}{12}$ (d) $\frac{2\pi}{3}$

8. Number of ordered pair (a, b) from the set $A = \{1, 2, 3, 4, 5\}$ so that the function $f(x) = \frac{x^3}{3} + \frac{a}{2}x^2 + bx + 10$ is an injective mapping $\forall x \in R$:

(a) 13 (b) 14 (c) 15 (d) 16

9. Let A be the greatest value of the function $f(x) = \log_x [x]$, (where $[\cdot]$ denotes greatest integer function) and B be the least value of the function $g(x) = |\sin x| + |\cos x|$, then :

(a) $A > B$ (b) $A < B$ (c) $A = B$ (d) $2A + B = 4$

10. Let $A = [a, \infty)$ denotes domain, then $f : [a, \infty) \to B, f(x) = 2x^3 - 3x^2 + 6$ will have an inverse for the smallest real value of a, if :

(a) $a = 1, B = [5, \infty)$ (b) $a = 2, B = [10, \infty)$ (c) $a = 0, B = [6, \infty)$ (d) $a = -1, B = [1, \infty)$

11. Solution of the inequation $\{x\}(\{x\} - 1)(\{x\} + 2) \geq 0$

(where $\{\cdot\}$ denotes fractional part function) is :

(a) $x \in (-2, 1)$ (b) $x \in I$ (I denote set of integers)

(c) $x \in [0, 1)$ (d) $x \in [-2, 0)$

12. Let $f(x), g(x)$ be two real valued functions then the function $h(x) = 2\max\{f(x) - g(x), 0\}$ is equal to :

(a) $f(x) - g(x) - |g(x) - f(x)|$ (b) $f(x) + g(x) - |g(x) - f(x)|$

(c) $f(x) - g(x) + |g(x) - f(x)|$ (d) $f(x) + g(x) + |g(x) - f(x)|$

13. Let $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ be a relation on the set $A = \{1, 2, 3, 4\}$. The relation R is :

(a) a function (b) reflexive (c) not symmetric (d) transitive

14. The true set of values of 'K' for which $\sin^{-1}\left(\frac{1}{1 + \sin^2 x}\right) = \frac{K\pi}{6}$ may have a solution is :

(a) $\left[\frac{1}{4}, \frac{1}{2}\right]$ (b) $[1, 3]$ (c) $\left[\frac{1}{6}, \frac{1}{2}\right]$ (d) $[2, 4]$

15. A real valued function $f(x)$ satisfies the functional equation $f(x - y) = f(x)f(y) - f(a - x)f(a + y)$ where 'a' is a given constant and $f(0) = 1$, $f(2a - x)$ is equal to :

(a) $-f(x)$ (b) $f(x)$ (c) $f(a) + f(a - x)$ (d) $f(-x)$

16. Let $g : R \to R$ be given by $g(x) = 3 + 4x$ if $g^n(x) = gogogo \ldots\ldots og(x)$ n times. Then inverse of $g^n(x)$ is equal to :

(a) $(x + 1 - 4^n) \cdot 4^{-n}$ (b) $(x - 1 + 4^n)4^{-n}$ (c) $(x + 1 + 4^n)4^{-n}$ (d) None of these

17. Let $f : D \to R$ be defined as : $f(x) = \frac{x^2 + 2x + a}{x^2 + 4x + 3a}$ where D and R denote the domain of f and the set of all real numbers respectively. If f is surjective mapping, then the complete range of a is :

(a) $0 \leq a \leq 1$ (b) $0 < a \leq 1$ (c) $0 \leq a < 1$ (d) $0 < a < 1$

18. If $f:(-\infty, 2] \longrightarrow (-\infty, 4]$, where $f(x) = x(4-x)$, then $f^{-1}(x)$ is given by :

(a) $2-\sqrt{4-x}$ (b) $2+\sqrt{4-x}$ (c) $-2+\sqrt{4-x}$ (d) $-2-\sqrt{4-x}$

19. If $[5\sin x] + [\cos x] + 6 = 0$, then range of $f(x) = \sqrt{3}\cos x + \sin x$ corresponding to solution set of the given equation is : (where $[\cdot]$ denotes greatest integer function)

(a) $[-2,-1)$ (b) $\left(-\frac{3\sqrt{3}+2}{5},-1\right)$ (c) $[-2,-\sqrt{3})$ (d) $\left(-\frac{3\sqrt{3}+4}{5},-1\right)$

20. If $f:R \to R$, $f(x) = ax + \cos x$ is an invertible function, then complete set of values of a is :

(a) $(-2,-1] \cup [1,2)$ (b) $[-1,1]$ (c) $(-\infty,-1] \cup [1,\infty)$ (d) $(-\infty,-2] \cup [2,\infty)$

21. The range of function $f(x) = [1+\sin x] + \left[2+\sin\frac{x}{2}\right] + \left[3+\sin\frac{x}{3}\right] + \ldots + \left[n+\sin\frac{x}{n}\right]\ \forall\, x \in [0,\pi]$, $n \in N$ ($[\cdot]$ denotes greatest integer function) is :

(a) $\left\{\frac{n^2+n-2}{2}, \frac{n(n+1)}{2}\right\}$ (b) $\left\{\frac{n(n+1)}{2}\right\}$

(c) $\left\{\frac{n(n+1)}{2}, \frac{n^2+n+2}{2}, \frac{n^2+n+4}{2}\right\}$ (d) $\left\{\frac{n(n+1)}{2}, \frac{n^2+n+2}{2}\right\}$

22. If $f:R \to R$, $f(x) = \frac{x^2+ax+1}{x^2+x+1}$, then the complete set of values of 'a' such that $f(x)$ is onto is :

(a) $(-\infty,\infty)$ (b) $(-\infty,0)$ (c) $(0,\infty)$ (d) not possible

23. If $f(x)$ and $g(x)$ are two functions such that $f(x) = [x] + [-x]$ and $g(x) = \{x\}\ \forall\, x \in R$ and $h(x) = f(g(x))$; then which of the following is incorrect ?

($[\cdot]$ denotes greatest integer function and $\{\cdot\}$ denotes fractional part function)

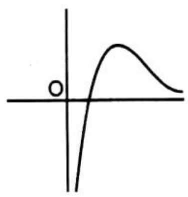
(a) $f(x)$ and $h(x)$ are identical functions (b) $f(x) = g(x)$ has no solution

(c) $f(x) + h(x) > 0$ has no solution (d) $f(x) - h(x)$ is a periodic function

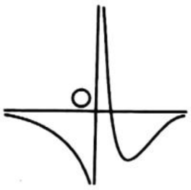
24. Number of elements in the range set of $f(x) = \left[\frac{x}{15}\right]\left[-\frac{15}{x}\right]\ \forall\, x \in (0, 90)$; (where $[\cdot]$ denotes greatest integer function) :

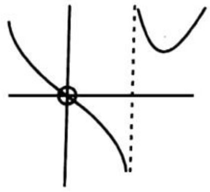
(a) 5 (b) 6 (c) 7 (d) Infinite

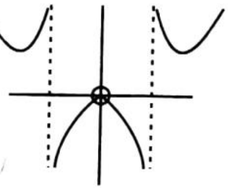
25. The graph of function $f(x)$ is shown below :

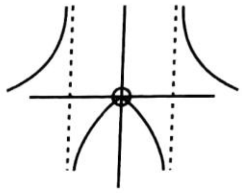


Then the graph of $g(x) = \frac{1}{f(|x|)}$ is :

(a) 

(b) 

(c) 

(d) 

26. Which of the following function is homogeneous ?

(a) $f(x) = x\sin y + y\sin x$

(b) $g(x) = xe^{\frac{y}{x}} + ye^{\frac{x}{y}}$

(c) $h(x) = \dfrac{xy}{x+y^2}$

(d) $\phi(x) = \dfrac{x - y\cos x}{y\sin x + y}$

27. Let $f(x) = \begin{bmatrix} 2x+3 & ; & x \le 1 \\ a^2x+1 & ; & x > 1 \end{bmatrix}$. If the range of $f(x) = R$ (set of real numbers) then number of integral value(s), which a may take :

(a) 2 (b) 3 (c) 4 (d) 5

28. The maximum integral value of x in the domain of $f(x) = \log_{10}(\log_{1/3}(\log_4(x-5)))$ is :

(a) 5 (b) 7 (c) 8 (d) 9

29. Range of the function $f(x) = \log_2\left(\dfrac{4}{\sqrt{x+2}+\sqrt{2-x}}\right)$ is :

(a) $(0, \infty)$ (b) $\left[\frac{1}{2}, 1\right]$ (c) $[1, 2]$ (d) $\left[\frac{1}{4}, 1\right]$

30. Number of integers statisfying the equation $|x^2+5x|+|x-x^2|=|6x|$ is :

(a) 3 (b) 5 (c) 7 (d) 9

31. Which of the following is not an odd function ?

(a) $\ln\left(\dfrac{x^4+x^2+1}{(x^2+x+1)^2}\right)$

(b) $\text{sgn}(\text{sgn}(x))$

(c) $\sin(\tan x)$

(d) $f(x)$, where $f(x)+f\left(\dfrac{1}{x}\right) = f(x)\cdot f\left(\dfrac{1}{x}\right)$ $\forall x \in R-\{0\}$ and $f(2) = 33$

32. Which of the following function is periodic with fundamental period π ?

(a) $f(x) = \cos x + \left[\left[\frac{\sin x}{2}\right]\right]$; where [·] denotes greatest integer function

(b) $g(x) = \frac{\sin x + \sin 7x}{\cos x + \cos 7x} + |\sin x|$

(c) $h(x) = \{x\} + |\cos x|$; where {·} denotes fractional part function

(d) $\phi(x) = |\cos x| + \ln(\sin x)$

33. Let $f: N \longrightarrow Z$ and $f(x) = \begin{cases} \frac{x-1}{2} & ; \text{ when } x \text{ is odd} \\ -\frac{x}{2} & ; \text{ when } x \text{ is even} \end{cases}$, then :

(a) $f(x)$ is bijective

(b) $f(x)$ is injective but not surjective

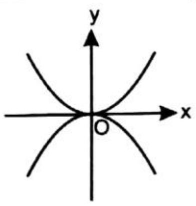
(c) $f(x)$ is not injective but surjective

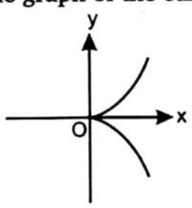
(d) $f(x)$ is neither injective nor surjective

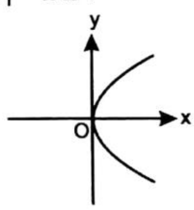
34. Let $g(x)$ be the inverse of $f(x) = \frac{2^{x+1} - 2^{1-x}}{2^x + 2^{-x}}$ then $g(x)$ be :

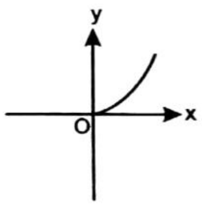
(a) $\frac{1}{2}\log_2\left(\frac{2+x}{2-x}\right)$ (b) $-\frac{1}{2}\log_2\left(\frac{2+x}{2-x}\right)$ (c) $\log_2\left(\frac{2+x}{2-x}\right)$ (d) $\log_2\left(\frac{2-x}{2+x}\right)$

35. Which of the following is the graph of the curve $\sqrt{|y|} = x$ is ?

(a)


(b)


(c)


(d)


36. Range of $f(x) = \log_{[x]}(9 - x^2)$; where [·] denotes G.I.F. is :

(a) {1, 2} (b) $(-\infty, 2)$ (c) $(-\infty, \log_2 5]$ (d) $[\log_2 5, 3]$

37. If $e^x + e^{f(x)} = e$, then for $f(x)$:

(a) Domain is $(-\infty, 1)$ (b) Range is $(-\infty, 1]$ (c) Domain is $(-\infty, 0]$ (d) Range is $(-\infty, 0]$

38. If high voltage current is applied on the field given by the graph $y + |y| - x - |x| = 0$. On which of the following curve a person can move so that he remains safe ?

(a) $y = x^2$ (b) $y = \text{sgn}(-e^2)$ (c) $y = \log_{1/3} x$ (d) $y = m + |x|; m > 3$

39. If $|f(x) + 6 - x^2] = |f(x)| + |4 - x^2| + 2$, then $f(x)$ is necessarily non-negaive for :

(a) $x \in [-2, 2]$

(b) $x \in (-\infty, -2) \cup (2, \infty)$

(c) $x \in [-\sqrt{6}, \sqrt{6}]$

(d) $x \in [-5, -2] \cup [2, 5]$

40. Let $f(x) = \cos(px) + \sin x$ be periodic, then p must be :

(a) Positive real number

(b) Negative real number

(c) Rational

(d) Prime

41. The domain of $f(x)$ is (0, 1), therefore, the domain of $y = f(e^x) + f(\ln|x|)$ is :

(a) $\left(\frac{1}{e}, 1\right)$ (b) $(-e, -1)$ (c) $\left(-1, -\frac{1}{e}\right)$ (d) $(-e, -1) \cup (1, e)$

42. Let $A = \{1, 2, 3, 4\}$ and $f : A \to A$ satisfy $f(1) = 2, f(2) = 3, f(3) = 4, f(4) = 1$.
Suppose $g : A \to A$ satisfies $g(1) = 3$ and $fog = gof$, then $g =$

(a) $\{(1, 3), (2, 1), (3, 2), (4, 4)\}$ (b) $\{(1, 3), (2, 4), (3, 1), (4, 2)\}$
(c) $\{(1, 3), (2, 2), (3, 4), (4, 3)\}$ (d) $\{(1, 3), (2, 4), (3, 2), (4, 1)\}$

43. The number of solutions of the equation $[y + [y]] = 2\cos x$ is :
(where $y = \frac{1}{3}[\sin x + [\sin x + [\sin x]]]$ and $[\cdot]$ = greatest integer function)

(a) 0 (b) 1 (c) 2 (d) Infinite

44. The function, $f(x) = \begin{cases} \dfrac{(x^{2n})}{(x^{2n}\,\text{sgn}\,x)^{2n+1}}\left(\dfrac{e^{\frac{1}{x}} - e^{-\frac{1}{x}}}{e^{\frac{1}{x}} + e^{-\frac{1}{x}}}\right) & x \neq 0 \\ 1 & x = 0 \end{cases}$ $n \in N$ is :

(a) Odd function (b) Even function
(c) Neither odd nor even function (d) Constant function

45. Let $f(1) = 1$, and $f(n) = 2\sum_{r=1}^{n-1} f(r)$. Then $\sum_{r=1}^{m} f(r)$ is equal to :

(a) $\frac{3^m - 1}{2}$ (b) 3^m (c) 3^{m-1} (d) $\frac{3^{m-1} - 1}{2}$

46. Let $f(x) = \frac{x}{\sqrt{1 + x^2}}$, then $\underbrace{fofofo\ldots\ldots of}_{n\text{times}}(x)$ is :

(a) $\dfrac{x}{\sqrt{1 + \left(\sum_{r=1}^{n} r\right)x^2}}$ (b) $\dfrac{x}{\sqrt{1 + \left(\sum_{r=1}^{n} 1\right)x^2}}$ (c) $\left(\dfrac{x}{\sqrt{1 + x^2}}\right)^n$ (d) $\dfrac{nx}{\sqrt{1 + nx^2}}$

47. Let $f : R \to R$, $f(x) = 2x + |\cos x|$, then f is :

(a) One-one and into (b) One-one and onto
(c) Many-one and into (d) Many-one and onto

48. Let $f : R \to R$, $f(x) = x^3 + x^2 + 3x + \sin x$, then f is :

(a) One-one and into (b) One-one and onto
(c) Many-one and into (d) Many-one and onto

49. $f(x) = \{x\} + \{x + 1\} + \{x + 2\} + \ldots\ldots + \{x + 99\}$, then $[f(\sqrt{2})]$, (where $\{\cdot\}$ denotes fractional part function and $[\cdot]$ denotes the greatest integer function) is equal to :

(a) 5050 (b) 4950 (c) 41 (d) 14

50. If $|\cot x + \operatorname{cosec} x| = |\cot x| + |\operatorname{cosec} x|$; $x \in [0, 2\pi]$, then complete set of values of x is :

(a) $[0, \pi]$

(b) $\left(0, \frac{\pi}{2}\right]$

(c) $\left(0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right)$

(d) $\left(\pi, \frac{3\pi}{2}\right] \cup \left[\frac{7\pi}{4}, 2\pi\right]$

51. The function $f(x) = 0$ has eight distinct real solution and f also satisfy $f(4+x) = f(4-x)$. The sum of all the eight solution of $f(x) = 0$ is :

(a) 12 (b) 32 (c) 16 (d) 15

52. Let $f(x)$ be a polynomial of degree 5 with leading coefficient unity such that $f(1) = 5, f(2) = 4,$ $f(3) = 3, f(4) = 2, f(5) = 1$. Then $f(6)$ is equal to :

(a) 0 (b) 24 (c) 120 (d) 720

53. Let $f : A \to B$ be a function such that $f(x) = \sqrt{x-2} + \sqrt{4-x}$, is invertible, then which of the following is not possible ?

(a) $A = [3, 4]$ (b) $A = [2, 3]$ (c) $A = [2, 2\sqrt{3}]$ (d) $[2, 2\sqrt{2}]$

54. The number of positive integral values of x satisfying $\left[\frac{x}{9}\right] = \left[\frac{x}{11}\right]$ is :

(where [·] denotes greatest integer function)

(a) 21 (b) 22 (c) 23 (d) 24

55. The domain of function $f(x) = \log_{\left[x+\frac{1}{2}\right]}(2x^2 + x - 1)$, where [·] denotes the greatest integer function is :

(a) $\left[\frac{3}{2}, \infty\right)$ (b) $(2, \infty)$ (c) $\left(-\frac{1}{2}, \infty\right) - \left\{\frac{1}{2}\right\}$ (d) $\left(\frac{1}{2}, 1\right) \cup (1, \infty)$

56. The solution set of the equation $[x]^2 + [x+1] - 3 = 0$, where [·] represents greatest integer function is :

(a) $[-1, 0) \cup [1, 2)$ (b) $[-2, -1) \cup [1, 2)$ (c) $[1, 2)$ (d) $[-3, -2) \cup [2, 3)$

57. Which among the following relations is a function ?

(a) $x^2 + y^2 = r^2$ (b) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = r^2$ (c) $y^2 = 4ax$ (d) $x^2 = 4ay$

(where a, b, r are constants)

58. A funciton $f : R \to R$ is defined as $f(x) = 3x^2 + 1$. Then $f^{-1}(x)$ is :

(a) $\frac{\sqrt{x-1}}{3}$

(b) $\frac{1}{3}\sqrt{x} - 1$

(c) f^{-1} does not exist

(d) $\sqrt{\frac{x-1}{3}}$

59. If $f(x)=\begin{cases}2+x, & x\ge 0\\ 4-x, & x<0\end{cases}$, then $f(f(x))$ is given by :

(a) $f(f(x))=\begin{cases}4+x, & x\ge 0\\ 6-x, & x<0\end{cases}$

(b) $f(f(x))=\begin{cases}4+x, & x\ge 0\\ x, & x<0\end{cases}$

(c) $f(f(x))=\begin{cases}4-x, & x\ge 0\\ x, & x<0\end{cases}$

(d) $f(f(x))=\begin{cases}4-2x, & x\ge 0\\ 4+2x, & x<0\end{cases}$

60. The function $f:R\to R$ defined as $f(x)=\dfrac{3x^2+3x-4}{3+3x-4x^2}$ is :

(a) One to one but not onto
(b) Onto but not one to one
(c) Both one to one and onto
(d) Neither one to one nor onto

61. The number of solutions of the equation $e^x-\log|x|=0$ is :

(a) 0 (b) 1 (c) 2 (d) 3

62. If complete solution set of $e^{-x}\le 4-x$ is $[\alpha,\beta]$, then $[\alpha]+[\beta]$ is equal to :
(where $[\cdot]$ denotes greatest integer function)

(a) 0 (b) 2 (c) 1 (d) 4

63. Range of $f(x)=\sqrt{\sin(\log_7(\cos(\sin x)))}$ is :

(a) $[0, 1)$ (b) $\{0, 1\}$ (c) $\{0\}$ (d) $[1, 7]$

64. If domain of $y=f(x)$ is $x\in[-3,2]$, then domain of $y=f(|[x]|)$:
(where $[\cdot]$ denotes greatest integer function)

(a) $[-3, 2]$ (b) $[-2, 3)$ (c) $[-3, 3]$ (d) $[-2, 3]$

65. Range of the function $f(x)=\cot^{-1}\{-x\}+\sin^{-1}\{x\}+\cos^{-1}\{x\}$, where $\{\cdot\}$ denotes fractional part function :

(a) $\left(\dfrac{3\pi}{4},\pi\right)$ (b) $\left[\dfrac{3\pi}{4},\pi\right)$ (c) $\left[\dfrac{3\pi}{4},\pi\right]$ (d) $\left(\dfrac{3\pi}{4},\pi\right]$

66. Let $f:R-\left\{\dfrac{3}{2}\right\}\to R$, $f(x)=\dfrac{3x+5}{2x-3}$. Let $f_1(x)=f(x)$, $f_n(x)=f(f_{n-1}(x))$ for $n\ge 2$, $n\in N$, then $f_{2008}(x)+f_{2009}(x)=$

(a) $\dfrac{2x^2+5}{2x-3}$ (b) $\dfrac{x^2+5}{2x-3}$ (c) $\dfrac{2x^2-5}{2x-3}$ (d) $\dfrac{x^2-5}{2x-3}$

67. Range of the function, $f(x)=\dfrac{(1+x+x^2)(1+x^4)}{x^3}$, for $x>0$ is :

(a) $[0,\infty)$ (b) $[2,\infty)$ (c) $[4,\infty)$ (d) $[6,\infty)$

68. The function $f:(-\infty,3]\to(0,e^7]$ defined by $f(x)=e^{x^3-3x^2-9x+2}$ is :

(a) Many-one and onto
(b) Many-one and into
(c) One to one and onto
(d) One to one and into

69. If $f(x) = \sin\left\{\log\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right\}; x \in R$, then range of $f(x)$ is given by :

(a) $[-1, 1]$ (b) $[0, 1]$ (c) $(-1, 1)$ (d) None of these

70. Set of values of 'a' for which the function $f : R \to R$, given by $f(x) = x^3 + (a+2)x^2 + 3ax + 10$ is one-one is given by :

(a) $(-\infty, 1] \cup [4, \infty)$ (b) $[1, 4]$ (c) $[1, \infty)$ (d) $[-\infty, 4]$

71. If the range of the function $f(x) = \tan^{-1}(3x^2 + bx + c)$ is $\left[0, \frac{\pi}{2}\right)$; (domain is R), then :

(a) $b^2 = 3c$ (b) $b^2 = 4c$ (c) $b^2 = 12c$ (d) $b^2 = 8c$

72. Let $f(x) = \sin^{-1} x - \cos^{-1} x$, then the set of values of k for which of $|f(x)| = k$ has exactly two distinct solutions is :

(a) $\left(0, \frac{\pi}{2}\right]$ (b) $\left(0, \frac{\pi}{2}\right)$ (c) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right)$ (d) $\left[\pi, \frac{3\pi}{2}\right]$

73. Let $f : R \to R$ is defined by $f(x) = \begin{cases} (x+1)^3 & ; x \le 1 \\ \ln x + (b^2 - 3b + 10) & ; x > 1 \end{cases}$. If $f(x)$ is invertible, then the set of all values of 'b' is :

(a) $\{1, 2\}$ (b) ϕ (c) $\{2, 5\}$ (d) None of these

74. Let $f(x)$ is continuous function with range $[-1, 1]$ and $f(x)$ is defined $\forall\ x \in R$. If $g(x) = \frac{e^{f(x)} - e^{|f(x)|}}{e^{f(x)} + e^{|f(x)|}}$, then range of $g(x)$ is :

(a) $[0, 1]$ (b) $\left[0, \frac{e^2+1}{e^2-1}\right]$

(c) $\left[0, \frac{e^2-1}{e^2+1}\right]$ (d) $\left[\frac{-e^2+1}{e^2+1}, 0\right]$

75. Consider all functions $f : \{1, 2, 3, 4\} \to \{1, 2, 3, 4\}$ which are one-one, onto and satisfy the following property :

if $f(k)$ is odd then $f(k+1)$ is even, $k = 1, 2, 3$.

The number of such functions is :

(a) 4 (b) 8 (c) 12 (d) 16

76. Consider the function $f : R - \{1\} \to R - \{2\}$ given by $f(x) = \frac{2x}{x-1}$. Then :

(a) f is one-one but not onto (b) f is onto but not one-one

(c) f is neither one-one nor onto (d) f is both one-one and onto

77. If range of function $f(x)$ whose domain is set of all real numbers is $[-2, 4]$, then range of function $g(x) = \frac{1}{2}f(2x+1)$ is equal to :

(a) $[-2, 4]$ (b) $[-1, 2]$ (c) $[-3, 9]$ (d) $[-2, 2]$

78. Let $f : R \to R$ and $f(x) = \dfrac{x(x^4+1)(x+1)+x^4+2}{x^2+x+1}$, then $f(x)$ is :

(a) One-one, into (b) Many-one, onto

(c) One-one, onto (d) Many one, into

79. Let $f(x)$ be defined as :

$$f(x) = \begin{cases} |x| & 0 \le x < 1 \\ |x-1| + |x-2| & 1 \le x < 2 \\ |x-3| & 2 \le x < 3 \end{cases}$$

The range of function $g(x) = \sin(7(f(x))$ is :

(a) $[0, 1]$ (b) $[-1, 0]$ (c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (d) $[-1, 1]$

80. If $[x]^2 - 7[x] + 10 < 0$ and $4[y]^2 - 16[y] + 7 < 0$, then $[x+y]$ cannot be ([·] denotes greatest integer function) :

(a) 7 (b) 8 (c) 9 (d) both (b) and (c)

81. Let $f : R \to R$ be a function defined by $f(x) = \dfrac{e^{|x|} - e^{-x}}{e^x + e^{-x}}$, then

(a) $f(x)$ is many one, onto function (b) $f(x)$ is many one, into function

(c) $f(x)$ is decreasing function $\forall x \in R$ (d) $f(x)$ is bijective function

82. The function $f(x)$ satisfy the equation $f(1-x) + 2f(x) = 3x \; \forall x \in R$, then $f(0) =$

(a) -2 (b) -1 (c) 0 (d) 1

83. Let $f : [0, 5] \to [0, 5]$ be an invertible function defined by $f(x) = ax^2 + bx + c$, where $a, b, c \in R$, $abc \neq 0$, then one of the root of the equation $cx^2 + bx + a = 0$ is :

(a) a (b) b (c) c (d) $a+b+c$

84. Let $f(x) = x^2 + \lambda x + \mu \cos x$, λ being an integer and μ is a real number. The number of ordered pairs (λ, μ) for which the equation $f(x) = 0$ and $f(f(x)) = 0$ have the same (non empty) set of real roots is :

(a) 2 (b) 3 (c) 4 (d) 6

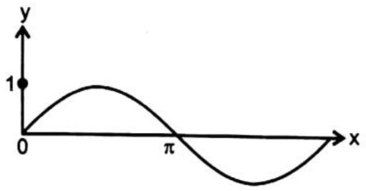
85. Consider all function $f : \{1, 2, 3, 4\} \to \{1, 2, 3, 4\}$ which are one-one, onto and satisfy the following property :

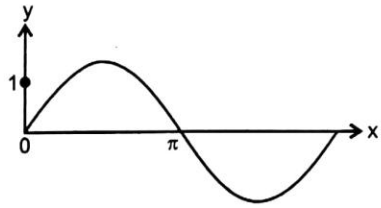
if $f(k)$ is odd then $f(k+1)$ is even, $k = 1, 2, 3$.

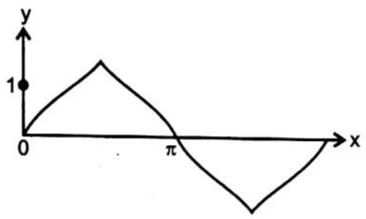
The number of such function is :

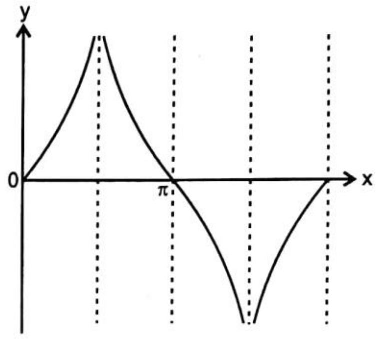
(a) 4 (b) 8 (c) 12 (d) 16

86. Which of the following is closest to the graph of $y = \tan(\sin x), x > 0$?

(a) 

(b) 

(c) 

(d) 

87. Consider the function $f: R - \{1\} \to R - \{2\}$ given by $f(x) = \dfrac{2x}{x-1}$. Then

(a) f is one-one but not onto
(b) f is onto but not one-one
(c) f is neither one-one nor onto
(d) f is both one-one and onto

88. If range of function $f(x)$ whose domain is set of all real numbers is $[-2, 4]$, then range of function $g(x) = \dfrac{1}{2} f(2x+1)$ is equal to :

(a) $[-2, 4]$ (b) $[-1, 2]$ (c) $[-3, 9]$ (d) $[-2, 2]$

89. Let $f: R \to R$ and $f(x) = \dfrac{x(x^4+1)(x+1)+x^4+2}{x^2+x+1}$, then $f(x)$ is :

(a) One-one, into (b) Many one, onto (c) One-one, onto (d) Many one, into

90. Let $f(x)$ be defined as

$$f(x) = \begin{cases} |x| & 0 \le x < 1 \\ |x-1| + |x-2| & 1 \le x < 2 \\ |x-3| & 2 \le x < 3 \end{cases}$$

The range of function $g(x) = \sin(7(f(x))$ is :

(a) $[0, 1]$ (b) $[-1, 0]$ (c) $\left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$ (d) $[-1, 1]$

91. The number of integral values of x in the domain of function f defined as $f(x) = \sqrt{\ln|\ln|x||} + \sqrt{7|x| - |x|^2 - 10}$ is :

(a) 5 (b) 6 (c) 7 (d) 8

92. The complete set of values of x in the domain of function $f(x)=\sqrt{\log_{x+2\{x\}}([x]^2-5[x]+7)}$ (where [·] denote greatest integer function and {·} denote fraction part function) is :

(a) $\left(-\frac{1}{3},0\right)\cup\left(\frac{1}{3},1\right)\cup(2,\infty)$ (b) $(0,1)\cup(1,\infty)$

(c) $\left(-\frac{2}{3},0\right)\cup\left(\frac{1}{3},1\right)\cup(1,\infty)$ (d) $\left(-\frac{1}{3},0\right)\cup\left(\frac{1}{3},1\right)\cup(1,\infty)$

93. The number of integral ordered pair (x,y) that satisfy the system of equation $|x+y-4|=5$ and $|x-3|+|y-1|=5$ is/are :

(a) 2 (b) 4 (c) 6 (d) 12

94. Let $f:R\to R$, where $f(x)=\frac{x^2+ax+1}{x^2+x+1}$. Then the complete set of values of 'a' such that $f(x)$ is onto is :

(a) $(-\infty,\infty)$ (b) $(-\infty,0)$ (c) $(0,\infty)$ (d) Empty set

95. If $A=\{1,2,3,4\}$ and $f:A\to A$, then total number of invertible function 'f' such that $f(2)\neq 2$, $f(4)\neq 4$, $f(1)=1$ is equal to :

(a) 1 (b) 2 (c) 3 (d) 4

96. The domain of definition of $f(x)=\log_{(x^2-x+1)}(2x^2-7x+9)$ is :

(a) R (b) $R-\{0\}$ (c) $R-\{0,1\}$ (d) $R-\{1\}$

97. If $A=\{1,2,3,4\}, B=\{1,2,3,4,5,6\}$ and $f:A\to B$ is an injective mapping satisfying $f(i)\neq i$, then number of such mappings are :

(a) 182 (b) 181 (c) 183 (d) none of these

98. Let $f(x)=x^2-2x-3; x\geq 1$ and $g(x)=1+\sqrt{x+4}; x\geq -4$ then the number of real solutions of equation $f(x)=g(x)$ is/are

(a) 0 (b) 1 (c) 2 (d) 4

Answers

1.	(b)	**2.**	(c)	**3.**	(c)	**4.**	(b)	**5.**	(d)	**6.**	(a)	**7.**	(d)	**8.**	(c)	**9.**	(c)	**10.**	(a)
11.	(b)	**12.**	(c)	**13.**	(c)	**14.**	(b)	**15.**	(a)	**16.**	(a)	**17.**	(d)	**18.**	(a)	**19.**	(d)	**20.**	(c)
21.	(d)	**22.**	(d)	**23.**	(b)	**24.**	(b)	**25.**	(c)	**26.**	(b)	**27.**	(c)	**28.**	(c)	**29.**	(b)	**30.**	(c)
31.	(d)	**32.**	(b)	**33.**	(a)	**34.**	(c)	**35.**	(b)	**36.**	(c)	**37.**	(a)	**38.**	(d)	**39.**	(a)	**40.**	(c)
41.	(b)	**42.**	(b)	**43.**	(a)	**44.**	(b)	**45.**	(c)	**46.**	(b)	**47.**	(b)	**48.**	(b)	**49.**	(c)	**50.**	(c)
51.	(b)	**52.**	(c)	**53.**	(c)	**54.**	(d)	**55.**	(a)	**56.**	(b)	**57.**	(d)	**58.**	(c)	**59.**	(a)	**60.**	(b)
61.	(b)	**62.**	(c)	**63.**	(c)	**64.**	(b)	**65.**	(d)	**66.**	(a)	**67.**	(d)	**68.**	(a)	**69.**	(a)	**70.**	(b)
71.	(c)	**72.**	(a)	**73.**	(a)	**74.**	(d)	**75.**	(c)	**76.**	(d)	**77.**	(b)	**78.**	(d)	**79.**	(d)	**80.**	(c)
81.	(b)	**82.**	(b)	**83.**	(a)	**84.**	(c)	**85.**	(c)	**86.**	(b)	**87.**	(d)	**88.**	(b)	**89.**	(d)	**90.**	(d)
91.	(b)	**92.**	(d)	**93.**	(d)	**94.**	(d)	**95.**	(c)	**96.**	(c)	**97.**	(b)	**98.**	(b)				

Exercise-2 : One or More than One Answer is/are Correct

1. $f(x)$ is an even periodic function with period 10. In $[0, 5]$, $f(x) = \begin{cases} 2x & 0 \le x < 2 \\ 3x^2 - 8 & 2 \le x < 4 \\ 10x & 4 \le x \le 5 \end{cases}$. Then :

(a) $f(-4) = 40$

(b) $\dfrac{f(-13) - f(11)}{f(13) + f(-11)} = \dfrac{17}{21}$

(c) $f(5)$ is not defined

(d) Range of $f(x)$ is $[0, 50]$

2. Let $f(x) = ||x^2 - 4x + 3| - 2|$. Which of the following is/are correct ?

(a) $f(x) = m$ has exactly two real solutions of different sign $\forall\, m > 2$

(b) $f(x) = m$ has exactly two real solutions $\forall\, m \in (2, \infty) \cup \{0\}$

(c) $f(x) = m$ has no solutions $\forall\, m < 0$

(d) $f(x) = m$ has four distinct real solution $\forall\, m \in (0, 1)$

3. Let $f(x) = \cos^{-1}\left(\dfrac{1 - \tan^2 (x/2)}{1 + \tan^2 (x/2)}\right)$

Which of the following statement(s) is/are correct about $f(x)$?

(a) Domain is R

(b) Range is $[0, \pi]$

(c) $f(x)$ is even

(d) $f(x)$ is derivable in $(\pi, 2\pi)$

4. $|\log_e |x|| = |k - 1| - 3$ has four distinct roots then k satisfies : (where $|x| < e^2, x \ne 0$)

(a) $(-4, -2)$

(b) $(4, 6)$

(c) (e^{-1}, e)

(d) (e^{-2}, e^{-1})

5. Which of the following functions are defined for all $x \in R$?
(Where $[\cdot]$ = denotes greatest integer function)

(a) $f(x) = \sin[x] + \cos[x]$

(b) $f(x) = \sec^{-1}(1 + \sin^2 x)$

(c) $f(x) = \sqrt{\dfrac{9}{8} + \cos x + \cos 2x}$

(d) $f(x) = \tan(\ln(1 + |x|))$

6. Let $f(x) = \begin{cases} x^2 & 0 < x < 2 \\ 2x - 3 & 2 \le x < 3 \\ x + 2 & x \ge 3 \end{cases}$, then the true equations :

(a) $f\left(f\left(f\left(\dfrac{3}{2}\right)\right)\right) = f\left(\dfrac{3}{2}\right)$

(b) $1 + f\left(f\left(f\left(\dfrac{5}{2}\right)\right)\right) = f\left(\dfrac{5}{2}\right)$

(c) $f(f(f(2))) = f(1)$

(d) $\underbrace{f(f(f(\ldots\ldots f(4))\ldots)}_{1004\text{ times}} = 2012$

7. Let $f : \left[\dfrac{2\pi}{3}, \dfrac{5\pi}{3}\right] \longrightarrow [0, 4]$ be a function defined as $f(x) = \sqrt{3} \sin x - \cos x + 2$, then :

(a) $f^{-1}(1) = \dfrac{4\pi}{3}$

(b) $f^{-1}(1) = \pi$

(c) $f^{-1}(2) = \dfrac{5\pi}{6}$

(d) $f^{-1}(2) = \dfrac{7\pi}{6}$

8. Let $f(x)$ be invertible function and let $f^{-1}(x)$ be its inverse. Let equation $f(f^{-1}(x)) = f^{-1}(x)$ has two real roots α and β (with in domain of $f(x)$), then :

(a) $f(x) = x$ also have same two real roots

(b) $f^{-1}(x) = x$ also have same two real roots

(c) $f(x) = f^{-1}(x)$ also have same two real roots

(d) Area of triangle formed by $(0, 0)$, $(\alpha, f(\alpha))$, and $(\beta, f(\beta))$ is 1 unit

9. The function $f(x) = \cos^{-1} x + \cos^{-1}\left(\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right)$, then :

(a) Range of $f(x)$ is $\left[\frac{\pi}{3}, \frac{10\pi}{3}\right]$

(b) Range of $f(x)$ is $\left[\frac{\pi}{3}, \frac{5\pi}{3}\right]$

(c) $f(x)$ is one-one for $x \in \left[-1, \frac{1}{2}\right]$

(d) $f(x)$ is one-one for $x \in \left[\frac{1}{2}, 1\right]$

10. Let $f: R \to R$ defined by $f(x) = \cos^{-1}(-\{-x\})$, where $\{x\}$ is fractional part function. Then which of the following is/are correct ?

(a) f is many-one but not even function

(b) Range of f contains two prime numbers

(c) f is a periodic

(d) Graph of f does not lie below x-axis

11. Which option(s) is/are true ?

(a) $f: R \to R$, $f(x) = e^{|x|} - e^{-x}$ is many-one into function

(b) $f: R \to R$, $f(x) = 2x + |\sin x|$ is one-one onto

(c) $f: R \to R$, $f(x) = \dfrac{x^2 + 4x + 30}{x^2 - 8x + 18}$ is many-one onto

(d) $f: R \to R$, $f(x) = \dfrac{2x^2 - x + 5}{7x^2 + 2x + 10}$ is many-one into

12. If $h(x) = \left[\ln\frac{x}{e}\right] + \left[\ln\frac{e}{x}\right]$, where $[\cdot]$ denotes greatest integer function, then which of the following are true ?

(a) range of $h(x)$ is $\{-1, 0\}$

(b) If $h(x) = 0$, then x must be irrational

(c) If $h(x) = -1$, then x can be rational as well as irrational

(d) $h(x)$ is periodic function

13. If $f(x) = \begin{cases} x^3 & ; \ x \in Q \\ -x^3 & ; \ x \notin Q \end{cases}$, then :

(a) $f(x)$ is periodic

(b) $f(x)$ is many-one

(c) $f(x)$ is one-one

(d) range of the function is R

14. Let $f(x)$ be a real valued continuous function such that

$f(0) = \frac{1}{2}$ and $f(x+y) = f(x)f(a-y) + f(y)f(a-x)\ \forall\ x, y \in R$,

then for some real a :

(a) $f(x)$ is a periodic function
(b) $f(x)$ is a constant function
(c) $f(x) = \frac{1}{2}$
(d) $f(x) = \frac{\cos x}{2}$

15. $f(x)$ is an even periodic function with period 10. In [0, 5], $f(x) = \begin{cases} 2x & 0 \le x < 2 \\ 3x^2 - 8 & 2 \le x < 4 \\ 10x & 4 \le x \le 5 \end{cases}$. Then :

(a) $f(-4) = 40$
(b) $\frac{f(-13) - f(11)}{f(13) + f(-11)} = \frac{17}{21}$
(c) $f(5)$ is not defined
(d) Range of $f(x)$ is [0, 50]

16. For the equation $\frac{e^{-x}}{1+x} = \lambda$ which of the following statement(s) is/are correct ?

(a) when $\lambda \in (0, \infty)$ equation has 2 real and distinct roots
(b) when $\lambda \in (-\infty, -e^2)$ equation has 2 real and distinct roots
(c) when $\lambda \in (0, \infty)$ equation has 1 real root
(d) when $\lambda \in (-e, 0)$ equation has no real root

17. For $x \in R^+$, if $x, [x], \{x\}$ are in harmonic progression then the value of x can not be equal to :
(where [·] denotes greatest integer function, {·} denotes fractional part function)

(a) $\frac{1}{\sqrt{2}} \tan \frac{\pi}{8}$
(b) $\frac{1}{\sqrt{2}} \cot \frac{\pi}{8}$
(c) $\frac{1}{\sqrt{2}} \tan \frac{\pi}{12}$
(d) $\frac{1}{\sqrt{2}} \cot \frac{\pi}{12}$

18. The equation $||x-1| + a| = 4$, $a \in R$, has :

(a) 3 distinct real roots for unique value of a.
(b) 4 distinct real roots for $a \in (-\infty, -4)$
(c) 2 distinct real roots for $|a| < 4$
(d) no real roots for $a > 4$

19. Let $f_n(x) = (\sin x)^{1/n} + (\cos x)^{1/n}$, $x \in R$, then :

(a) $f_2(x) > 1$ for all $x \in \left(2k\pi, (4k+1)\frac{\pi}{2}\right)$, $k \in I$

(b) $f_2(x) = 1$ for $x = 2k\pi$, $k \in I$

(c) $f_2(x) > f_3(x)$ for all $x \in \left(2k\pi, (4k+1)\frac{\pi}{2}\right)$, $k \in I$

(d) $f_3(x) \ge f_5(x)$ for all $x \in \left(2k\pi, (4k+1)\frac{\pi}{2}\right)$, $k \in I$

(Where I denotes set of integers)

20. If the domain of $f(x) = \frac{1}{\pi}\cos^{-1}\left[\log_3\left(\frac{x^2}{3}\right)\right]$ where, $x > 0$ is $[a, b]$ and the range of $f(x)$ is $[c, d]$, then :

(a) a, b are the roots of the equation $x^4 - 3x^3 - x + 3 = 0$
(b) a, b are the roots of the equation $x^4 - x^3 + x^2 - 2x + 1 = 0$
(c) $a^3 + d^3 = 1$
(d) $a^2 + b^2 + c^2 + d^2 = 11$

21. The number of real values of x satisfying the equation ; $\left[\frac{2x+1}{3}\right] + \left[\frac{4x+5}{6}\right] = \frac{3x-1}{2}$ are greater than or equal to {[·] denotes greatest integer function):

(a) 7 (b) 8 (c) 9 (d) 10

22. Let $f(x) = \sin^6\left(\frac{x}{4}\right) + \cos^6\left(\frac{x}{4}\right)$. If $f^n(x)$ denotes n^{th} derivative of f evaluated at x. Then which of the following hold ?

(a) $f^{2014}(0) = -\frac{3}{8}$ (b) $f^{2015}(0) = \frac{3}{8}$ (c) $f^{2010}\left(\frac{\pi}{2}\right) = 0$ (d) $f^{2011}\left(\frac{\pi}{2}\right) = \frac{3}{8}$

23. Which of the following is(are) incorrect ?

(a) If $f(x) = \sin x$ and $g(x) = \ln x$ then range of $g(f(x))$ is $[-1, 1]$
(b) If $x^2 + ax + 9 > x \,\forall\, x \in R$ then $-5 < a < 7$
(c) If $f(x) = (2011 - x^{2012})^{\frac{1}{2012}}$ then $f(f(2)) = \frac{1}{2}$
(d) The function $f: R \to R$ defined as $f(x) = \frac{x^2 + 4x + 30}{x^2 - 8x + 18}$ is not surjective.

24. If $[x]$ denotes the integral part of x for real x, and

$$S = \left[\frac{1}{4}\right] + \left[\frac{1}{4} + \frac{1}{200}\right] + \left[\frac{1}{4} + \frac{1}{100}\right] + \left[\frac{1}{4} + \frac{3}{200}\right] \ldots\ldots + \left[\frac{1}{4} + \frac{199}{200}\right]$$ then

(a) S is a composite number
(b) Exponent of S in $\lfloor 100$ is 12
(c) Number of factors of S is 10
(d) ${}^{2S}C_r$ is max when $r = 51$

Answers

1.	(a, b, d)	**2.**	(a, b, c)	**3.**	(c, d)	**4.**	(a, b)	**5.**	(a, b, c)	**6.**	(a, b, c, d)
7.	(a, d)	**8.**	(a, b, c)	**9.**	(b, c)	**10.**	(a, b, d)	**11.**	(a, b, d)	**12.**	(a, c)
13.	(c, d)	**14.**	(a, b, c)	**15.**	(a, b, d)	**16.**	(b, c, d)	**17.**	(a, c, d)	**18.**	(a, b, c, d)
19.	(a, b)	**20.**	(a, d)	**21.**	(a, b, c)	**22.**	(a, c, d)	**23.**	a, b)	**24.**	(a, b)

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 3

Let $f(x) = \log_{\{x\}}[x]$

$g(x) = \log_{\{x\}}\{x\}$

$h(x) \log_{[x]}\{x\}$

where [ ], { } denotes the greatest integer function and fractional part fucntion respectively.

1. For $x \in (1, 5)$ the $f(x)$ is not defined at how many points :

(a) 5 (b) 4 (c) 3 (d) 2

2. If $A = \{x : x \in$ domain of $f(x)\}$ and $B = \{x : x \in$ domain of $g(x)\}$ then $\forall\ x \in (1, 5)$, $A - B$ will be :

(a) (2, 3) (b) (1, 3) (c) (1, 2) (d) None of these

3. Domain of $h(x)$ is :

(a) $[2, \infty)$ (b) $[1, \infty)$ (c) $[2, \infty) - \{I\}$ (d) $R^+ - \{I\}$

I denotes integers.

Paragraph for Question Nos. 4 to 6

θ is said to be well behaved if it lies in interval $\left[0, \frac{\pi}{2}\right]$. They are intelligent if they make domain of $f + g$ and g equal. The values of θ for which $h(\theta)$ is defined are handsome. Let

$$f(x) = \sqrt{\theta x^2 - 2(\theta^2 - 3)x - 12\theta}, g(x) = \ln(x^2 - 49),$$

$$h(\theta) = \ln\left[\int_0^{\theta} 4\cos^2 t\, dt - \theta^2\right], \text{ where } \theta \text{ is in radians.}$$

4. Complete set of values of θ which are well behaved as well as intelligent is :

(a) $\left[\frac{3}{4}, \frac{\pi}{2}\right]$ (b) $\left[\frac{3}{5}, \frac{7}{8}\right]$ (c) $\left[\frac{5}{6}, \frac{\pi}{2}\right]$ (d) $\left[\frac{6}{7}, \frac{\pi}{2}\right]$

5. Complete set of values of θ which are intelligent is :

(a) $\left[\frac{6}{7}, \frac{7}{2}\right]$ (b) $\left(0, \frac{\pi}{3}\right]$ (c) $\left[\frac{1}{4}, \frac{6}{7}\right]$ (d) $\left[\frac{1}{2}, \frac{\pi}{2}\right]$

6. Complete set of values of θ which are well behaved, intelligent and handsome is :

(a) $\left(0, \frac{\pi}{2}\right]$ (b) $\left[\frac{6}{7}, \frac{\pi}{2}\right]$ (c) $\left[\frac{3}{4}, \frac{\pi}{2}\right]$ (d) $\left[\frac{3}{5}, \frac{\pi}{2}\right]$

Paragraph for Question Nos. 7 to 8

Let $f(x) = 2 - |x-3|, 1 \le x \le 5$ and for rest of the values $f(x)$ can be obtained by using the relation $f(5x) = \alpha f(x) \; \forall x \in R$.

7. The maximum value of $f(x)$ in $[5^4, 5^5]$ for $\alpha = 2$ is :

(a) 16 (b) 32 (c) 64 (d) 8

8. The value of $f(2007)$, taking $\alpha = 5$, is :

(a) 1118 (b) 2007 (c) 1250 (d) 132

Paragraph for Question Nos. 9 to 10

An even periodic function $f : R \to R$ with period 4 is such that

$$f(x) = \begin{cases} \max.(|x|, x^2) & ; \; 0 \le x < 1 \\ x & ; \; 1 \le x \le 2 \end{cases}$$

9. The value of $\{f(5.12)\}$ (where $\{\cdot\}$ denotes fractional part function), is :

(a) $\{f(3.26)\}$ (b) $\{f(7.88)\}$ (c) $\{f(2.12)\}$ (d) $\{f(5.88)\}$

10. The number of solutions of $f(x) = |3\sin x|$ for $x \in (-6, 6)$ are :

(a) 5 (b) 3 (c) 7 (d) 9

Paragraph for Question Nos. 11 to 12

Let $f(x) = \dfrac{2|x|-1}{x-3}$

11. Range of $f(x)$:

(a) $R - \{3\}$ (b) $\left(-\infty, \frac{1}{3}\right] \cup (2, \infty)$ (c) $\left(-2, \frac{1}{3}\right] \cup (2, \infty)$ (d) R

12. Range of the values of 'k' for which $f(x) = k$ has exactly two distinct solutions :

(a) $\left(-2, \frac{1}{3}\right)$ (b) $(-2, 1]$ (c) $\left(0, \frac{2}{3}\right]$ (d) $(-\infty, -2)$

Paragraph for Question Nos. 13 to 14

Let $f(x)$ be a continuous function (define for all x) which satisfies $f^3(x) - 5f^2(x) + 10f(x) - 12 \ge 0$, $f^2(x) - 4f(x) + 3 \ge 0$ and $f^2(x) - 5f(x) + 6 \le 0$

13. If distinct positive number b_1, b_2 and b_3 are in G.P. then $f(1) + \ln b_1$, $f(2) + \ln b_2$, $f(3) + \ln b_3$ are in :

(a) A.P. (b) G.P. (c) H.P. (d) A.G.P.

14. The equation of tangent that can be drawn from (2, 0) on the curve $y = x^2 f(\sin x)$ is :

(a) $y = 24(x+2)$ (b) $y = 12(x+2)$ (c) $y = 24(x-2)$ (d) $y = 12(x-2)$

Paragraph for Question Nos. 15 to 16

Let $f:[2,\infty) \to [1,\infty)$ defined by $f(x) = 2^{x^4-4x^2}$ and $g:\left[\frac{\pi}{2},\pi\right] \to A$ defined by $g(x) = \frac{\sin x + 4}{\sin x - 2}$ be two invertible functions, then

15. $f^{-1}(x)$ is equal to

(a) $\sqrt{2+\sqrt{4-\log_2 x}}$ (b) $\sqrt{2+\sqrt{4+\log_2 x}}$ (c) $\sqrt{4+\sqrt{2+\log_2 x}}$ (d) $\sqrt{4-\sqrt{2+\log_2 x}}$

16. The set 'A' equals to

(a) $[5, 2]$ (b) $[-2, 5]$ (c) $[-5, 2]$ (d) $[-5, -2]$

Answers

1.	(c)	**2.**	(d)	**3.**	(c)	**4.**	(d)	**5.**	(a)	**6.**	(b)	**7.**	(b)	**8.**	(a)	**9.**	(b)	**10.**	(c)
11.	(b)	**12.**	(a)	**13.**	(a)	**14.**	(c)	**15.**	(b)	**16.**	(d)								

Exercise-4 : Matching Type Problems

1. If $x, y, z \in R$ satisfies the system of equations $x + [y] + \{z\} = 12.7$, $[x] + \{y\} + z = 4.1$ and $\{x\} + y + [z] = 2$
(where $\{\cdot\}$ and $[\cdot]$ denotes the fractional and integral parts respectively), then match the following :

	Column-I		Column-II
(A)	$\{x\} + \{y\} =$	**(P)**	7.7
(B)	$[z] + [x] =$	**(Q)**	1.1
(C)	$x + \{z\} =$	**(R)**	1
(D)	$z + [y] - \{x\} =$	**(S)**	3
		(T)	4

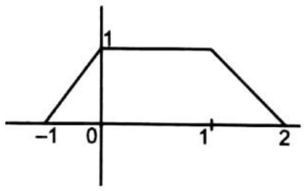
2. Consider $ax^4 + (7a - 2b)x^3 + (12a - 14b - c)x^2 - (24b + 7c)x + 1 - 12c = 0$, has no real roots

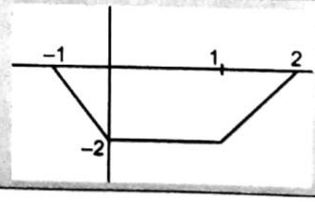
and $f_1(x) = \dfrac{\sqrt{\log_{(\pi+e)}(ax^4 + (7a - 2b)x^3 + (12a - 14b - c)x^2 - (24b + 7c)x + 1 - 12c)}}{\sqrt{a}\sqrt{-\operatorname{sgn}(1 + ac + b^2)}}$

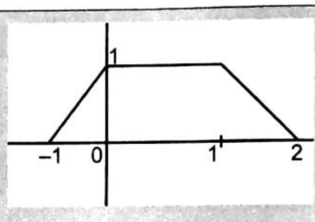
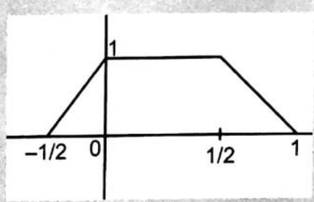
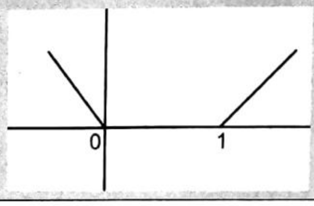
$f_2(x) = -2 + 2\log_{\sqrt{2}} \cos\left(\tan^{-1}(\sin(\pi(\cos(\pi(x + \frac{7}{2})))))\right)$. Then match the following :

	Column-I		Column-II
(A)	Domain of $f_1(x)$ is	**(P)**	$[-3, -2]$
(B)	Range of $f_2(x)$ in the domain of $f_1(x)$ is	**(Q)**	$[-4, -2]$
(C)	Range of $f_2(x)$ is	**(R)**	$(-\infty, \infty)$
(D)	Domain of $f_2(x)$ is	**(S)**	$(-\infty, -4] \cup [-3, \infty)$
		(T)	$[0, 1]$

3. Given the graph of $y = f(x)$



	Column-I		Column-II
(A)	$y = f(1 - x)$	**(P)**	

(B)	$y = f(2x)$	**(Q)**	
(C)	$y = -2f(x)$	**(R)**	
(D)	$y = 1 - f(x)$	**(S)**	

4.

	Column-I		Column-II
(A)	$f(x) = \sin^2 2x - 2\sin^2 x$	**(P)**	Range contains no natural number
(B)	$f(x) = \dfrac{4}{\pi}(\sin^{-1}(\sin \pi x))$	**(Q)**	Range contains atleast one integer
(C)	$f(x) = \sqrt{\ln(\cos(\sin x))}$	**(R)**	Many one but not even function
(D)	$f(x) = \tan^{-1}\left(\dfrac{x^2+1}{x^2+\sqrt{3}}\right)$	**(S)**	Both many one and even function
		(T)	Periodic but not odd function

5.

	Column-I		Column-II
(A)	If $\lvert x^2 - x\rvert \geq x^2 + x$, then complete set of values of x is	**(P)**	$(0, \infty)$
(B)	If $\lvert x + y\rvert > x - y$, where $x > 0$, then complete set of values of y is	**(Q)**	$(-\infty, 0]$
(C)	If $\log_2 x \geq \log_2(x^2)$, then complete set of values of x is	**(R)**	$[-1, \infty)$

(D)	$[x] + 2 \geq \lvert x \rvert$, (where $[\cdot]$ denotes the greatest integer function) then complete set of values of x is	**(S)**	$(0, 1]$
		(T)	$[1, \infty)$

6.

	Column-I		Column-II
(A)	Domain of $f(x) = \ln \tan^{-1}\{(x^3 - 6x^2 + 11x - 6)\, x(e^x - 1)\}$ is	**(P)**	$\left[-1, \frac{5}{4}\right]$
(B)	Range of $f(x) = \sin^2 \frac{x}{4} + \cos \frac{x}{4}$ is	**(Q)**	$[2, \infty)$
(C)	The domain of function $f(x) = \sqrt{\log_{(\lvert x \rvert - 1)}(x^2 + 4x + 4)}$ is	**(R)**	$(1, 2) \cup (3, \infty)$
(D)	Let $f(x) = \begin{cases} x^2 & x < 1 \\ x + 1 & x \geq 1 \end{cases}$; $g(x) = \begin{cases} x + 2 & x < 1 \\ x^2 & x \geq 1 \end{cases}$	**(S)**	$[0, \infty)$
	Then range of function $f(g(x))$ is	**(T)**	$(-\infty, -3) \cup (-2, -1) \cup (2, \infty)$

7. Let $f(x) \begin{bmatrix} 1 + x; \ 0 \leq x \leq 2 \\ 3 - x; \ 2 < x \leq 3 \end{bmatrix}$;

$g(x) = f(f(x))$:

	Column-I		Column-II
(A)	If domain of $g(x)$ is $[a, b]$ then $b - a$ is	**(P)**	1
(B)	If range of $g(x)$ is $[c, d]$ then $c + d$ is	**(Q)**	2
(C)	$f(f(f(2))) + f(f(f(3)))$, is	**(R)**	3
(D)	m = maximum value of $g(x)$ then $2m - 2$ is :	**(S)**	4

Answers

1. A → R ; B → S ; C → P ; D → Q

2. A → S ; B → P ; C → Q ; D → R

3. A → Q ; B → R ; C → P ; D → S

4. A → P, Q, S, T ; B → Q, R ; C → P, Q, S ; D → P, S

5. A → Q; B → P; C → S; D → R

6. A → R; B → P; C → T; D → S

7. A → R ; B → R ; C → R ; D → S

Exercise-5 : Subjective Type Problems

1. Let $f(x)$ be a polynomial of degree 6 with leading coefficient 2009. Suppose further, that $f(1)=1, f(2)=3, f(3)=5, f(4)=7, f(5)=9, f'(2)=2$, then the sum of all the digits of $f(6)$ is

2. Let $f(x)=x^3-3x+1$. Find the number of different real solution of the equation $f(f(x))=0$.

3. If $f(x+y+1)=(\sqrt{f(x)}+\sqrt{f(y)})^2\ \forall x, y \in R$ and $f(0)=1$, then $f(2)=......$

4. If the domain of $f(x)=\sqrt{12-3^x-3^{3-x}}+\sin^{-1}\left(\frac{2x}{3}\right)$ is $[a, b]$, then $a=......$

5. The number of elements in the range of the function :

$y=\sin^{-1}\left[x^2+\frac{5}{9}\right]+\cos^{-1}\left[x^2-\frac{4}{9}\right]$ where $[\cdot]$ denotes the greatest integer function is

6. The number of solutions of the equation $f(x-1)+f(x+1)=\sin\alpha,\ 0<\alpha<\frac{\pi}{2}$, where $f(x)=\begin{cases}1-|x| & , |x|\le 1\\ 0 & , |x|>1\end{cases}$ is

7. The number of integers in the range of function $f(x)=[\sin x]+[\cos x]+[\sin x+\cos x]$ is (where $[\cdot]$ = denotes greatest integer function)

8. If $P(x)$ is a polynomial of degree 4 such that $P(-1)=P(1)=5$ and $P(-2)=P(0)=P(2)=2$, then find the maximum value of $P(x)$.

9. The number of integral value(s) of k for which the curve $y=\sqrt{-x^2-2x}$ and $x+y-k=0$ intersect at 2 distinct points is/are

10. Let the solution set of the equation :

$$\sqrt{\left[x+\left[\frac{x}{2}\right]\right]}+\left[\sqrt{\{x\}}+\left[\frac{x}{3}\right]\right]=3$$

is $[a, b)$. Find the product ab.

(where $[\cdot]$ and $\{\cdot\}$ denote greatest integer and fractional part function respectively).

11. For all real number x, let $f(x)=\frac{1}{\sqrt[2011]{1-x^{2011}}}$. Find the number of real roots of the equation

$$f(f(......(f(x))......))=\{-x\}$$

where f is applied 2013 times and $\{\cdot\}$ denotes fractional part function.

12. Find the number of elements contained in the range of the function $f(x)=\left[\frac{x}{6}\right]\left[\frac{-6}{x}\right]\ \forall x \in (0, 30]$ (where $[\cdot]$ denotes greatest integer function)

13. Let $f(x, y)=x^2-y^2$ and $g(x, y)=2xy$.

such that $(f(x, y))^2-(g(x, y))^2=\frac{1}{2}$ and $f(x, y)\cdot g(x, y)=\frac{\sqrt{3}}{4}$

Find the number of ordered pairs (x, y) ?

14. Let $f(x)=\dfrac{x+5}{\sqrt{x^2+1}}\ \forall\ x\in R$, then the smallest integral value of k for which $f(x)\le k\ \forall\ x\in R$ is

15. In the above problem, $f(x)$ is injective in the interval $x\in(-\infty,a]$, and λ is the largest possible value of a, then $[\lambda]=$
(where $[x]$ denote greatest integer $\le x$)

16. The number of integral values of m for which $f:R\to R;\ f(x)=\dfrac{x^3}{3}+(m-1)x^2+(m+5)x+n$ is bijective is :

17. The number of roots of equation :

$$\left(\frac{(x-1)(x-3)}{(x-2)(x-4)}-e^x\right)\left(\frac{(x+1)(x+3)e^x}{(x+2)(x+4)}-1\right)(x^3-\cos x)=0$$

18. The number of solutions of the equation $\cos^{-1}\left(\dfrac{1-x^2-2x}{(x+1)^2}\right)=\pi(1-\{x\})$, for $x\in[0,76]$ is equal to. (where $\{\cdot\}$ denote fraction part function)

19. Let $f(x)=x^2-bx+c$, b is an odd positive integer. Given that $f(x)=0$ has two prime numbers as roots and $b+c=35$. If the least value of $f(x)\ \forall\ x\in R$ is λ, then $\left[\left|\dfrac{\lambda}{3}\right|\right]$ is equal to
(where $[\cdot]$ denotes greatest integer function)

20. Let $f(x)$ be continuous function such that $f(0)=1$ and $f(x)-f\left(\dfrac{x}{7}\right)=\dfrac{x}{7}\ \forall\ x\in R$, then $f(42)=$

21. If $f(x)=4x^3-x^2-2x+1$ and $g(x)=\begin{cases}\min\{f(t):0\le t\le x\} & ;\ 0\le x\le 1\\ 3-x & ;\ 1<x\le 2\end{cases}$ and if $\lambda=g\left(\dfrac{1}{4}\right)+g\left(\dfrac{3}{4}\right)+g\left(\dfrac{5}{4}\right)$, then $2\lambda=$

22. If $x=10\displaystyle\sum_{r=3}^{100}\frac{1}{(r^2-4)}$, then $[x]=$
(where $[\cdot]$ denotes greatest integer function)

23. Let $f(x)=\dfrac{ax+b}{cx+d}$, where a,b,c,d are non zero. If $f(7)=7$, $f(11)=11$ and $f(f(x))=x$ for all x except $-\dfrac{d}{c}$. The unique number which is not in the range of f is

24. Let $A=\{x\mid x^2-4x+3<0,\ x\in R\}$
$B=\{x\mid 2^{1-x}+p\le 0;\ x^2-2(p+7)x+5\le 0\}$
If $A\subseteq B$, then the range of real number $p\in[a,b]$ where a,b are integers.
Find the value of $(b-a)$.

25. Let the maximum value of expression $y = \dfrac{x^4 - x^2}{x^6 + 2x^3 - 1}$ for $x > 1$ is $\dfrac{p}{q}$, where p and q are relatively prime natural numbers, then $p + q =$

26. If $f(x)$ is an even function, then the number of distinct real numbers x such that $f(x) = f\left(\dfrac{x+1}{x+2}\right)$ is :

27. The least integral value of $m, m \in R$ for which the range of function $f(x) = \dfrac{x+m}{x^2+1}$ contains the interval $[0, 1]$ is :

28. Let x_1, x_2, x_3 satisfying the equation $x^3 - x^2 + \beta x + \gamma = 0$ are in G.P. where x_1, x_2, x_3 are positive numbers. Then the maximum value of $[\beta] + [\gamma] + 4$ is where $[\cdot]$ denotes greatest integer function is :

29. Let $A = \{1, 2, 3, 4\}$ and $B = \{0, 1, 2, 3, 4, 5\}$. If 'm' is the number of strictly increasing function f, $f: A \to B$ and n is the number of onto functions $g, g: B \to A$. Then the last digit of $n - m$ is.

30. If $\sum_{r=1}^{n} [\log_2 r] = 2010$, where $[\cdot]$ denotes greatest integer function, then the sum of the digits of n is :

31. Let $f(x) = \dfrac{ax+b}{cx+d}$, where a, b, c, d are non-zero. If $f(7) = 7$, $f(11) = 11$ and $f(f(x)) = x$ for all x except $-\dfrac{d}{c}$. The unique number which is not in the range of f is

32. It is pouring down rain, and the amount of rain hitting point (x, y) is given by $f(x, y) = | x^3 + 2x^2y - 5xy^2 - 6y^3 |$. If Mr. '$A$' starts at $(0, 0)$; find number of possible value(s) for 'm' such that $y = mx$ is a line along which Mr. 'A' could walk without any rain falling on him.

33. Let $P(x)$ be a cubic polynomical with leading co-efficient unity. Let the remainder when $P(x)$ is divided by $x^2 - 5x + 6$ equals 2 times the remainder when $P(x)$ is divided by $x^2 - 5x + 4$. If $P(0) = 100$, find the sum of the digits of $P(5)$:

34. Let $f(x) = x^2 + 10x + 20$. Find the number of real solution of the equation $f(f(f(f(x)))) = 0$

35. If range of $f(x) = \dfrac{(\ln x)(\ln x^2) + \ln x^3 + 3}{\ln^2 x + \ln x^2 + 2}$ can be expressed as $\left[\dfrac{a}{b}, \dfrac{c}{d}\right]$ where a, b, c and d are prime numbers (not necessarily distinct) then find the value of $\dfrac{(a+b+c+d)}{2}$.

36. Polynomial $P(x)$ contains only terms of odd degree. When $P(x)$ is divided by $(x-3)$, then remainder is 6. If $P(x)$ is divided by $(x^2 - 9)$ then remainder is $g(x)$. Find the value of $g(2)$.

37. The equation $2x^3 - 3x^2 + p = 0$ has three real roots. Then find the minimum value of p.

38. Find the number of integers in the domain of $f(x) = \dfrac{1}{\sqrt{\ln \cos^{-1} x}}$.

Answers

1.	26	**2.**	7	**3.**	9	**4.**	1	**5.**	1	**6.**	4	**7.**	5
8.	6	**9.**	1	**10.**	12	**11.**	1	**12.**	6	**13.**	4	**14.**	6
15.	0	**16.**	6	**17.**	7	**18.**	76	**19.**	6	**20.**	8	**21.**	5
22.	5	**23.**	9	**24.**	3	**25.**	7	**26.**	4	**27.**	1	**28.**	3
29.	5	**30.**	8	**31.**	9	**32.**	3	**33.**	2	**34.**	2	**35.**	6
36.	4	**37.**	0	**38.**	2								

❑❑❑

Chapter 2 – Limit

2

LIMIT

Exercise-1 : Single Choice Problems

1. $\lim_{x\to 0} \frac{\cos(\tan x) - \cos x}{x^4} =$

(a) $\frac{1}{6}$ (b) $-\frac{1}{3}$ (c) $-\frac{1}{6}$ (d) $\frac{1}{3}$

2. The value of $\lim_{x\to 0} \frac{(\sin x - \tan x)^2 - (1-\cos 2x)^4 + x^5}{7(\tan^{-1} x)^7 + (\sin^{-1} x)^6 + 3\sin^5 x}$ equal to :

(a) 0 (b) 1 (c) 2 (d) $\frac{1}{3}$

3. Let $a = \lim_{x\to 0} \frac{\ln(\cos 2x)}{3x^2}$, $b = \lim_{x\to 0} \frac{\sin^2 2x}{x(1-e^x)}$, $c = \lim_{x\to 1} \frac{\sqrt{x} - x}{\ln x}$.

Then a, b, c satisfy :

(a) $a < b < c$ (b) $b < c < a$ (c) $a < c < b$ (d) $b < a < c$

4. If $f(x) = \cot^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$ and $g(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, then $\lim_{x\to a} \frac{f(x) - f(a)}{g(x) - g(a)}$, $0 < a < \frac{1}{2}$ is :

(a) $\frac{3}{2(1+a^2)}$ (b) $\frac{3}{2}$ (c) $\frac{-3}{2(1+a^2)}$ (d) $-\frac{3}{2}$

5. $\lim_{x\to 0} \left(\frac{(1+x)^{\frac{2}{x}}}{e^2}\right)^{\frac{4}{\sin x}}$ is :

(a) e^4 (b) e^{-4} (c) e^8 (d) e^{-8}

6. $\lim_{x\to\infty} \frac{3}{x}\left[\frac{x}{4}\right] = \frac{p}{q}$ (where [·] denotes greatest integer function), then $p + q$ (where p, q are relative prime) is :

(a) 2 (b) 7 (c) 5 (d) 6

7. $f(x) = \lim_{n\to\infty} \dfrac{x^n + \left(\dfrac{\pi}{3}\right)^n}{x^{n-1} + \left(\dfrac{\pi}{3}\right)^{n-1}}$, ($n$ is an even integer), then which of the following is incorrect ?

(a) If $f:\left[\dfrac{\pi}{3}, \infty\right) \to \left[\dfrac{\pi}{3}, \infty\right)$, then function is invertible

(b) $f(x) = f(-x)$ has infinite number of solutions

(c) $f(x) = |f(x)|$ has infinite number of solutions

(d) $f(x)$ is one-one function for all $x \in R$

8. $\lim_{x\to 0} \dfrac{\sin(\pi\cos^2(\tan(\sin x)))}{x^2} =$

(a) π (b) $\dfrac{\pi}{4}$ (c) $\dfrac{\pi}{2}$ (d) none of these

9. If $f(x) = \begin{cases} \dfrac{(e^{(x+3)\ln 27})^{\frac{x}{27}} - 9}{3^x - 27} & ; \; x < 3 \\ \lambda\dfrac{1-\cos(x-3)}{(x-3)\tan(x-3)} & ; \; x > 3 \end{cases}$

If $\lim_{x\to 3} f(x)$ exist, then $\lambda =$

(a) $\dfrac{9}{2}$ (b) $\dfrac{2}{9}$ (c) $\dfrac{2}{3}$ (d) none of these

10. $\lim_{x\to \frac{\pi}{3}} \dfrac{\sin\left(\dfrac{\pi}{3} - x\right)}{2\cos x - 1}$ is equal to :

(a) $\dfrac{2}{\sqrt{3}}$ (b) $\dfrac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $\dfrac{1}{2}$

11. $\lim_{x\to \frac{\pi}{2}} \dfrac{\sin x}{\cos^{-1}\left[\dfrac{1}{4}(3\sin x - \sin 3x)\right]}$, (where [·] denotes greatest integer function) is :

(a) $\dfrac{2}{\pi}$ (b) 1 (c) $\dfrac{4}{\pi}$ (d) does not exist

12. Let f be a continuous function on R such that $f\left(\dfrac{1}{4^n}\right) = (\sin e^n)e^{-n^2} + \dfrac{n^2}{n^2+1}$, then $f(0) =$

(a) 1 (b) 0 (c) -1 (d) $\dfrac{1}{4}$

13. $\lim_{x \to I^-} \dfrac{e^{\{x\}} - \{x\} - 1}{\{x\}^2}$ equals, where $\{\cdot\}$ is fractional part function and I is an integer, to :

(a) $\dfrac{I}{2}$ (b) $e - 2$ (c) I (d) does not exist

14. $\lim_{x \to \infty} (e^{11x} - 7x)^{\frac{1}{3x}}$ is equal to :

(a) $\dfrac{11}{3}$ (b) $\dfrac{3}{11}$ (c) $e^{\frac{3}{11}}$ (d) $e^{\frac{11}{3}}$

15. The value of $\lim_{x \to 0} \left[(1-2x)^n \sum_{r=0}^{n} {}^nC_r \left(\dfrac{x + x^2}{1-2x} \right)^r \right]^{1/x}$ is :

(a) e^n (b) e^{-n} (c) e^{3n} (d) e^{-3n}

16. For a certain value of 'c', $\lim_{x \to \infty} [(x^5 + 7x^4 + 2)^c - x]$ is finite and non-zero. Then the value of limit is :

(a) $\dfrac{7}{5}$ (b) 1 (c) $\dfrac{2}{5}$ (d) None of these

17. The number of non-negative integral values of n for which $\lim_{x \to 0} \dfrac{(\cos x - 1)(\cos x - e^x)}{x^n} = 0$ is :

(a) 1 (b) 2 (c) 3 (d) 4

18. The value of $\lim_{x \to 0} \left(\dfrac{\sin x}{x} \right)^{\frac{1}{1-\cos x}}$:

(a) $e^{-1/3}$ (b) $e^{1/3}$ (c) $e^{-1/6}$ (d) $e^{1/6}$

19. If $\lim_{x \to \infty} (\sqrt{x^2 - x + 1} - ax - b) = 0$, then for $k \geq 2, (k \in N)$ $\lim_{n \to \infty} \sec^{2n}(k!\pi b) =$

(a) a (b) $-a$ (c) $2a$ (d) b

20. If f is a positive function such that $f(x+T) = f(x)\,(T > 0)$, $\forall\, x \in R$, then

$$\lim_{n \to \infty} n \left(\frac{f(x+T) + 2f(x+2T) + \ldots\ldots + nf(x+nT)}{f(x+T) + 4f(x+4T) + \ldots\ldots + n^2 f(x+n^2T)} \right) =$$

(a) 2 (b) $\dfrac{2}{3}$ (c) $\dfrac{3}{2}$ (d) None of these

21. Let $f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$

$$265 \left(\lim_{h \to 0} \frac{h^4 + 3h^2}{(f(1-h) - f(1)) \sin 5h} \right) =$$

(a) 1 (b) 2 (c) 3 (d) -3

22. $\lim_{x\to 0}\left(\dfrac{\cos x-\sec x}{x^2(x+1)}\right)=$

(a) 0 (b) $-\dfrac{1}{2}$ (c) -1 (d) -2

23. Let $f(x)$ be a continuous and differentiable function satisfying $f(x+y)=f(x)f(y)\ \forall\ x,y\in R$ if $f(x)$ can be expressed as $f(x)=1+xP(x)+x^2Q(x)$ where $\lim_{x\to 0}P(x)=a$ and $\lim_{x\to 0}Q(x)=b$, then $f'(x)$ is equal to :

(a) $a\,f(x)$ (b) $b\,f(x)$

(c) $(a+b)\,f(x)$ (d) $(a+2b)\,f(x)$

24. $\lim_{x\to\frac{\pi}{2}}\dfrac{\left(1-\tan\dfrac{x}{2}\right)(1-\sin x)}{\left(1+\tan\dfrac{x}{2}\right)(\pi-2x)^3}=$

(a) not exist (b) $\dfrac{1}{8}$ (c) $\dfrac{1}{16}$ (d) $\dfrac{1}{32}$

25. $\lim_{x\to\infty}\left(\dfrac{x-3}{x+2}\right)^x$ is equal to :

(a) e (b) e^{-1} (c) e^{-5} (d) e^5

26. $\lim_{x\to\frac{\pi}{2}}(\cos x)^{\cos x}$ is :

(a) 1 (b) 0 (c) $\dfrac{1}{e}$ (d) $\dfrac{2}{e}$

27. If $\lim_{x\to c^-}\{\ln x\}$ and $\lim_{x\to c^+}\{\ln x\}$ exists finitely but they are not equal (where $\{\cdot\}$ denotes fractional part function), then :

(a) 'c' can take only rational values

(b) 'c' can take only irrational values

(c) 'c' can take infinite values in which only one is irrational

(d) 'c' can take infinite values in which only one is rational

28. $\lim_{x\to 0}\left(1+\dfrac{a\sin bx}{\cos x}\right)^{\frac{1}{x}}$, where a, b are non-zero constants is equal to :

(a) $e^{a/b}$ (b) ab

(c) e^{ab} (d) $e^{b/a}$

29. The value of $\lim_{x\to 0}\left((\cos x)^{\frac{1}{\sin^2 x}}+\frac{\sin 2x+2\tan^{-1}3x+3x^2}{\ln(1+3x+\sin^2 x)+xe^x}\right)$ is :

(a) $\sqrt{e}+\frac{3}{2}$ (b) $\frac{1}{\sqrt{e}}+\frac{3}{2}$ (c) $\sqrt{e}+2$ (d) $\frac{1}{\sqrt{e}}+2$

30. Let $a=\lim_{x\to 1}\left(\frac{x}{\ln x}-\frac{1}{x\ln x}\right)$; $b=\lim_{x\to 0}\frac{x^3-16x}{4x+x^2}$; $c=\lim_{x\to 0}\frac{\ln(1+\sin x)}{x}$ and

$d=\lim_{x\to -1}\frac{(x+1)^3}{3[\sin(x+1)-(x+1)]}$, then the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is :

(a) Idempotent (b) Involutary
(c) Non-singular (d) Nilpotent

31. The integral value of n so that $\lim_{x\to 0} f(x)$ where $f(x)=\frac{(\sin x-x)\left(2\sin x-\ln\left(\frac{1+x}{1-x}\right)\right)}{x^n}$ is a finite non-zero number, is :

(a) 2 (b) 4 (c) 6 (d) 8

32. Consider the function $f(x)=\begin{cases}\dfrac{\max\left(x,\frac{1}{x}\right)}{\min\left(x,\frac{1}{x}\right)}, & \text{if } x\neq 0\\ 1, & \text{if } x=0\end{cases}$, then $\lim_{x\to 0^-}\{f(x)\}+\lim_{x\to 1^-}\{f(x)\}+\lim_{x\to -1^-}[f(x)]=$

(where $\{\cdot\}$ denotes fraction part function and $[\cdot]$ denotes greatest integer function)

(a) 0 (b) 1 (c) 2 (d) 3

33. $\lim_{x\to\left(\frac{1}{\sqrt{2}}\right)^+}\frac{\cos^{-1}(2x\sqrt{1-x^2})}{\left(x-\frac{1}{\sqrt{2}}\right)}-\lim_{x\to\left(\frac{1}{\sqrt{2}}\right)^-}\frac{\cos^{-1}(2x\sqrt{1-x^2})}{\left(x-\frac{1}{\sqrt{2}}\right)}=$

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $4\sqrt{2}$ (d) 0

34. $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\sin\frac{\pi}{2k}-\cos\frac{\pi}{2k}-\sin\left(\frac{\pi}{2(k+2)}\right)+\cos\frac{\pi}{2(k+2)}\right)=$

(a) 0 (b) 1
(c) 2 (d) 3

35. $\lim_{x\to 0^+}[1+[x]]^{2/x}$, where $[\cdot]$ is greatest integer function, is equal to :

(a) 0 (b) 1
(c) e^2 (d) Does not exist

36. If m and n are positive integers, then $\lim_{x\to 0} \frac{(\cos x)^{1/m} - (\cos x)^{1/n}}{x^2}$ equals to :

(a) $m-n$

(b) $\frac{1}{n}-\frac{1}{m}$

(c) $\frac{m-n}{2mn}$

(d) None of these

37. The value of ordered pair (a, b) such that $\lim_{x\to 0} \frac{x(1+a\cos x)-b\sin x}{x^3} = 1$, is :

(a) $\left(-\frac{5}{2}, -\frac{3}{2}\right)$ (b) $\left(\frac{5}{2}, \frac{3}{2}\right)$ (c) $\left(-\frac{5}{2}, \frac{3}{2}\right)$ (d) $\left(\frac{5}{2}, -\frac{3}{2}\right)$

38. What is the value of $a+b$, if $\lim_{x\to 0} \frac{\sin(ax) - \ln(e^x \cos x)}{x\sin(bx)} = \frac{1}{2}$?

(a) 1 (b) 2 (c) 3 (d) $-\frac{1}{2}$

39. Let $\alpha = \lim_{n\to\infty} \frac{(1^3-1^2)+(2^3-2^2)+\ldots+(n^3-n^2)}{n^4}$, then α is equal to :

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) non existent

40. The value of $\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4}$ is equal to :

(a) $\frac{1}{5}$ (b) $\frac{1}{6}$ (c) $\frac{1}{4}$ (d) $\frac{1}{12}$

41. The value of ordered pair (a, b) such that $\lim_{x\to 0} \frac{x(1+a\cos x)-b\sin x}{x^3} = 1$, is :

(a) $\left(-\frac{5}{2}, -\frac{3}{2}\right)$ (b) $\left(\frac{5}{2}, \frac{3}{2}\right)$ (c) $\left(-\frac{5}{2}, \frac{3}{2}\right)$ (d) $\left(\frac{5}{2}, -\frac{3}{2}\right)$

42. Consider the sequence :

$$u_n = \sum_{r=1}^{n} \frac{r}{2^r}, \quad n \geq 1$$

Then the limit of u_n as $n \to \infty$ is :

(a) 1 (b) e (c) $\frac{1}{2}$ (d) 2

43. The value of $\lim_{x\to 0} \left((\cos x)^{\frac{1}{\sin^2 x}} + \frac{\sin 2x + 2\tan^{-1} 3x + 3x^2}{\ln(1+3x+\sin^2 x) + xe^x} \right)$ is :

(a) $\sqrt{e}+\frac{3}{2}$ (b) $\frac{1}{\sqrt{e}}+\frac{3}{2}$ (c) $\sqrt{e}+2$ (d) $\frac{1}{\sqrt{e}}+2$

44. For $n \in N$, let $f_n(x) = \tan\frac{x}{2}(1+\sec x)(1+\sec 2x)(1+\sec 4x)\ldots\ldots(1+\sec 2^n x)$, the $\lim_{x\to 0}\frac{f_n(x)}{2x}$ is equal to :

(a) 0 (b) 2^n (c) 2^{n-1} (d) 2^{n+1}

45. The value of $\lim_{x\to\frac{\pi}{4}}(1+[x])^{\frac{1}{ln(\tan x)}}$ is :

(where [.] denotes greatest integer function).

(a) 0 (b) 1 (c) e (d) $\frac{1}{e}$

46. If $\lim_{x\to 0}\frac{\{(a-n)nx-\tan x\}\sin nx}{x^2}=0$, $n \neq 0$ then a is equal to :

(a) 0 (b) $1+\frac{1}{n}$ (c) n (d) $n+\frac{1}{n}$

47. The value of $\lim_{n\to\infty}\left(\frac{n!}{n^n}\right)^{\frac{3n^3+4}{4n^4-1}}$, $n \in N$ is equal to :

(a) $\left(\frac{1}{e}\right)^{3/4}$ (b) $e^{3/4}$ (c) e^{-1} (d) 0

48. The value of $\lim_{x\to\infty}\frac{ax^2+bx+c}{dx+e}$ $(a, b, c, d, e \in R-\{0\})$ depends on the sign of :

(a) a only (b) d only
(c) a and d only (d) a, b and d only

49. Let $f(x)=\lim_{n\to\infty}\tan^{-1}\left(4n^2\left(1-\cos\frac{x}{n}\right)\right)$ and $g(x)=\lim_{n\to\infty}\frac{n^2}{2}\ln\cos\left(\frac{2x}{n}\right)$ then $\lim_{x\to 0}\frac{e^{-2g(x)}-e^{f(x)}}{x^6}$ equals.

(a) $\frac{8}{3}$ (b) $\frac{7}{3}$ (c) $\frac{5}{3}$ (d) $\frac{2}{3}$

50. If $f(x)$ be a cubic polynomial and $\lim_{x\to 0}\frac{\sin^2 x}{f(x)}=\frac{1}{3}$ then $f(1)$ can not be equal to :

(a) 0 (b) -5 (c) 3 (d) -2

51. $\lim_{x\to 0}\frac{2e^{\sin x}-e^{-\sin x}-1}{x^2+2x}$ equals to :

(a) $\frac{3}{2}$ (b) $e^{3/2}$ (c) 2 (d) e^2

52. If $x_1, x_2, x_3, \ldots\ldots, x_n$ are the roots of $x^n + ax + b = 0$, then the value of $(x_1 - x_2)(x_1 - x_3)$ $(x_1 - x_4)\ldots\ldots(x_1 - x_n)$ is equal to :

(a) $nx_1 + b$ (b) $nx_1^{n-1} + a$ (c) nx_1^{n-1} (d) nx_1^{n-1}

53. $\lim_{x\to 0} \frac{\sqrt[3]{1+\sin^2 x} - \sqrt[4]{1-2\tan x}}{\sin x + \tan^2 x}$ is equal to :

(a) -1 (b) 1 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

54. If $f(x) = \begin{vmatrix} x\cos x & 2x\sin x & x\tan x \\ 1 & x & 1 \\ 1 & 2x & 1 \end{vmatrix}$, find $\lim_{x\to 0} \frac{f(x)}{x^2}$.

(a) 0 (b) 1 (c) -1 (d) Does not exist

Answers

1.	(b)	**2.**	(d)	**3.**	(d)	**4.**	(d)	**5.**	(b)	**6.**	(b)	**7.**	(d)	**8.**	(a)	**9.**	(c)	**10.**	(b)
11.	(a)	**12.**	(a)	**13.**	(b)	**14.**	(d)	**15.**	(b)	**16.**	(a)	**17.**	(c)	**18.**	(a)	**19.**	(a)	**20.**	(c)
21.	(c)	**22.**	(c)	**23.**	(a)	**24.**	(d)	**25.**	(c)	**26.**	(a)	**27.**	(d)	**28.**	(c)	**29.**	(d)	**30.**	(d)
31.	(c)	**32.**	(a)	**33.**	(c)	**34.**	(d)	**35.**	(b)	**36.**	(c)	**37.**	(a)	**38.**	(b)	**39.**	(b)	**40.**	(b)
41.	(a)	**42.**	(d)	**43.**	(d)	**44.**	(c)	**45.**	(b)	**46.**	(d)	**47.**	(a)	**48.**	(c)	**49.**	(a)	**50.**	(c)
51.	(a)	**52.**	(b)	**53.**	(c)	**54.**	(c)												

Exercise-2 : One or More than One Answer is/are Correct

1. If $\lim\limits_{x\to 0}(p\tan qx^2 - 3\cos^2 x + 4)^{1/(3x^2)} = e^{5/3}$; $p, q \in R$ then :

(a) $p=\sqrt{2}, q=\dfrac{1}{2\sqrt{2}}$ (b) $p=\dfrac{1}{\sqrt{2}}, q=2\sqrt{2}$ (c) $p=1, q=2$ (d) $p=2, q=4$

2. $\lim\limits_{x\to\infty} 2\left(\sqrt{25x^2+x}-5x\right)$ is equal to :

(a) $\lim\limits_{x\to 0}\dfrac{2x-\log_e(1+x)^2}{5x^2}$ (b) $\lim\limits_{x\to 0}\dfrac{e^{-x}-1+x}{x^2}$

(c) $\lim\limits_{x\to 0}\dfrac{2(1-\cos x^2)}{5x^4}$ (d) $\lim\limits_{x\to 0}\dfrac{\sin\frac{x}{5}}{x}$

3. Let $\lim\limits_{x\to\infty}(2^x+a^x+e^x)^{1/x}=L$

which of the following statement(s) is(are) correct ?

(a) if $L=a\,(a>0)$, then the range of a is $[e,\infty)$

(b) if $L=2e(a>0)$, then the range of a is $\{2e\}$

(c) if $L=e\,(a>0)$, then the range of a is $(0, e]$

(d) if $L=2a\,(a>1)$, then the range of a is $\left(\dfrac{e}{2},\infty\right)$

4. Let $\tan\alpha\cdot x+\sin\alpha\cdot y=\alpha$ and $\alpha\operatorname{cosec}\alpha\cdot x+\cos\alpha\cdot y=1$ be two variable straight lines, α being the parameter. Let P be the point of intersection of the lines. In the limiting position when $\alpha\to 0$, the point P lies on the line :

(a) $x=2$ (b) $x=-1$ (c) $y+1=0$ (d) $y=2$

5. Let $f:R\to[-1,1]$ be defined as $f(x)=\cos(\sin x)$, then which of the following is(are) correct ?

(a) f is periodic with fundamental period 2π (b) Range of $f=[\cos 1, 1]$

(c) $\lim\limits_{x\to\frac{\pi}{2}}\left(f\left(\dfrac{\pi}{2}-x\right)+f\left(\dfrac{\pi}{2}+x\right)\right)=2$ (d) f is neither even nor odd function

6. Let $f(x)=x+\sqrt{x^2+2x}$ and $g(x)=\sqrt{x^2+2x}-x$, then :

(a) $\lim\limits_{x\to\infty}g(x)=1$ (b) $\lim\limits_{x\to\infty}f(x)=1$ (c) $\lim\limits_{x\to-\infty}f(x)=-1$ (d) $\lim\limits_{x\to\infty}g(x)=-1$

7. Which of the following limits does not exist ?

(a) $\lim\limits_{x\to\infty}\operatorname{cosec}^{-1}\left(\dfrac{x}{x+7}\right)$ (b) $\lim\limits_{x\to 1}\sec^{-1}(\sin^{-1}x)$

(c) $\lim\limits_{x\to 0^+}x^{\frac{1}{x}}$ (d) $\lim\limits_{x\to 0}\left(\tan\left(\dfrac{\pi}{8}+x\right)\right)^{\cot x}$

8. If $f(x) = \lim_{n\to\infty} x\left(\frac{3}{2} + [\cos x]\left(\sqrt{n^2+1} - \sqrt{n^2-3n+1}\right)\right)$ where $[y]$ denotes largest integer $\le y$, then identify the correct statement(s).

(a) $\lim_{x\to 0} f(x) = 0$

(b) $\lim_{x\to \frac{\pi}{2}} f(x) = \frac{3\pi}{4}$

(c) $f(x) = \frac{3x}{2} \forall x \in \left[0, \frac{\pi}{2}\right]$

(d) $f(x) = 0 \forall x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

9. Let $f: R \to R;\ f(x) = \begin{cases} (-1)^n & \text{if } x = \frac{1}{2^{2^n}},\quad n = 1, 2, 3....... \\ 0 & \text{otherwise} \end{cases}$

then identify the correct statement(s).

(a) $\lim_{x\to 0} f(x) = 0$

(b) $\lim_{x\to 0} f(x)$ does not exist

(c) $\lim_{x\to 0} f(x) f(2x) = 0$

(d) $\lim_{x\to 0} f(x) f(2x)$ does not exist

10. If $\lim_{x\to a} f(x) = \lim_{x\to a} [f(x)]$ ([·] denotes the greatest integer function) and $f(x)$ is non-constant continuous function, then :

(a) $\lim_{x\to a} f(x)$ is an integer

(b) $\lim_{x\to a} f(x)$ is non-integer

(c) $f(x)$ has local maximum at $x = a$

(d) $f(x)$ has local minimum at $x = a$

11. Let $f(x) = \frac{\cos^{-1}(1-\{x\})\sin^{-1}(1-\{x\})}{\sqrt{2\{x\}}(1-\{x\})}$ where $\{x\}$ denotes the fractional part of x, then :

(a) $\lim_{x\to 0^+} f(x) = \frac{\pi}{4}$

(b) $\lim_{x\to 0^+} f(x) = \sqrt{2} \lim_{x\to 0^-} f(x)$

(c) $\lim_{x\to 0^-} f(x) = \frac{\pi}{4\sqrt{2}}$

(d) $\lim_{x\to 0^-} f(x) = \frac{\pi}{2\sqrt{2}}$

12. If $\lim_{x\to 0} \frac{(\sin(\sin x) - \sin x)}{ax^3 + bx^5 + c} = -\frac{1}{12}$, then:

(a) $a = 2$ (b) $a = -2$ (c) $c = 0$ (d) $b \in R$

13. If $f(x) = \lim_{n\to\infty} (n(x^{1/n} - 1))$ for $x > 0$, then which of the following is/are true ?

(a) $f\left(\frac{1}{x}\right) = 0$

(b) $f\left(\frac{1}{x}\right) = \frac{1}{f(x)}$

(c) $f\left(\frac{1}{x}\right) = -f(x)$

(d) $f(xy) = f(x) + f(y)$

14. The value of $\lim\limits_{n\to\infty} \cos^2 (\pi (\sqrt[3]{n^3 + n^2 + 2n}))$ (where $n \in N$):

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{9}$

15. If $\alpha, \beta \in \left(-\frac{\pi}{2}, 0\right)$ such that $(\sin\alpha + \sin\beta) + \frac{\sin\alpha}{\sin\beta} = 0$ and $(\sin\alpha + \sin\beta)\ \frac{\sin\alpha}{\sin\beta} = -1$ and

$\lambda = \lim\limits_{n\to\infty} \frac{1 + (2\sin\alpha)^{2n}}{(2\sin\beta)^{2n}}$ then :

(a) $a = -\frac{\pi}{6}$ (b) $\lambda = 2$ (c) $\alpha = -\frac{\pi}{3}$ (d) $\lambda = 1$

16. Let $f(x) = \begin{cases} |x-2| + a^2 - 6a + 9 & , \quad x < 2 \\ 5 - 2x & , \quad x \geq 2 \end{cases}$

If $\lim\limits_{x\to 2} [f(x)]$ exists, the possible values a can take is/are (where [·] represents the greatest integer function)

(a) 2 (b) $\frac{5}{2}$ (c) 3 (d) $\frac{7}{2}$

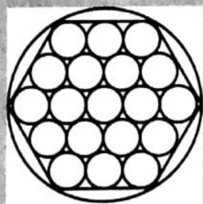
Answers

1.	(b, c)	**2.**	(a, c, d)	**3.**	(a, b, c)	**4.**	(a, c)	**5.**	(b, c)	**6.**	(a, c)
7.	(a, d)	**8.**	(a, c, d)	**9.**	(b, c)	**10.**	(a, d)	**11.**	(b, d)	**12.**	(a, c)
13.	(c, d)	**14.**	(c)	**15.**	(a, b)	**16.**	(b)				

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

A circular disk of unit radius is filled with a number of smaller circular disks arranged in the form of hexagon. Let A_n denotes a stack of disks arranged in the shape of a hexagon having 'n' disks on a side. The figure shows the configuration A_3. If 'A' be the area of large disk, S_n be the number of disks in A_n configuration and r_n be the radius of each disk in A_n configuration, then



A_3 Configuration

1. $\lim\limits_{n\to\infty} \frac{S_n}{n^2}$:

(a) 3 (b) 4 (c) 1 (d) 11

2. $\lim\limits_{n\to\infty} nr_n$:

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{11}$

Paragraph for Question Nos. 3 to 4

Let $f(x) = \begin{bmatrix} x+3 & ; & -2 < x < 0 \\ 4 & ; & x = 0 \\ 2x+5 & ; & 0 < x < 1 \end{bmatrix}$, then

3. $\lim\limits_{x\to 0^-} f([x - \tan x])$ is : ([·] denotes greatest integer fucntion)

(a) 2 (b) 4 (c) 5 (d) None of these

4. $\lim\limits_{x\to 0^+} f\left(\left\{\frac{x}{\tan x}\right\}\right)$ is : ({·} denotes fractional part of function)

(a) 4 (b) 5 (c) 7 (d) None of these

Paragraph for Question Nos. 5 to 6

A certain function $f(x)$ has the property that $f(3x) = \alpha f(x)$ for all positive real values of x and $f(x) = 1 - |x-2|$ for $1 \le x \le 3$.

5. $\lim\limits_{x\to 2} (f(x))^{\operatorname{cosec}\left(\frac{\pi x}{2}\right)}$ is :

(a) $\frac{2}{\pi}$ (b) $-\frac{2}{\pi}$

(c) $e^{2/\pi}$ (d) Non-existent

6. If the total area bounded by $y = f(x)$ and x-axis in $[1, \infty)$ converges to a finite quantity, then the range of α is :

(a) $(-1, 1)$ (b) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ (c) $\left(-\frac{1}{3}, \frac{1}{3}\right)$ (d) $\left(-\frac{1}{4}, \frac{1}{4}\right)$

Paragraph for Question Nos. 7 to 9

Consider the limit $\lim_{x \to 0} \frac{1}{x^3}\left(\frac{1}{\sqrt{1+x}} - \frac{(1+ax)}{(1+bx)}\right)$ exists, finite and has the value equal to l (where a, b are real constants), then :

7. $a =$

(a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

8. $a + b =$

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) 0

9. $\left|\frac{b}{l}\right| =$

(a) 38 (b) 16 (c) 72 (d) 24

Paragraph for Question Nos. 10 to 11

For the curve $\sin x + \sin y = 1$ lying in the first quadrant there exists a constant α for which

$$\lim_{x \to 0} x^{\alpha} \frac{d^2y}{dx^2} = L \text{ (not zero)}$$

10. The value of α :

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{3}{2}$ (d) 2

11. The value of L :

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{1}{2\sqrt{2}}$ (d) $\frac{1}{2\sqrt{3}}$

Answers

1.	(a)	**2.**	(b)	**3.**	(b)	**4.**	(c)	**5.**	(d)	**6.**	(c)	**7.**	(d)	**8.**	(c)	**9.**	(d)	**10.**	(c)
11.	(c)																		

Exercise-4 : Matching Type Problems

1.

	Column-I		Column-II
(A)	$\lim\limits_{n\to\infty}\left(\dfrac{1+\sqrt[n]{4}}{2}\right)^n =$	**(P)**	2
(B)	Let $f(x)=\lim\limits_{n\to\infty}\dfrac{2x}{\pi}\tan^{-1}(nx)$, then $\lim\limits_{x\to 0^+} f(x) =$	**(Q)**	0
(C)	$\lim\limits_{x\to \frac{\pi}{2}^+}\dfrac{\cos(\tan^{-1}(\tan x))}{x-\frac{\pi}{2}} =$	**(R)**	1
(D)	If $\lim\limits_{x\to 0^+}(x)^{\frac{1}{\ln \sin x}} = e^L$, then $L+2=$	**(S)**	3
		(T)	Non-existent

2. [·] represents greatest integer function :

	Column-I		Column-II
(A)	If $f(x)=\sin^{-1}x$ and $\lim\limits_{x\to\frac{1}{2}^+} f(3x-4x^3) = a-3\lim\limits_{x\to\frac{1}{2}^-} f(x)$, then $[a]=$	**(P)**	2
(B)	If $f(x)=\tan^{-1}g(x)$ where $g(x)=\dfrac{3x-x^3}{1-3x^2}$ and then find $\left[\lim\limits_{h\to 0}\dfrac{f\left(\frac{1}{2}+6h\right)-f\left(\frac{1}{2}\right)}{6h}\right]=$	**(Q)**	3
(C)	If $\cos^{-1}(4x^3-3x) = a+b\cos^{-1}x$ for $-1<x<\dfrac{-1}{2}$, then $[a+b+2]=$	**(R)**	4
(D)	If $f(x)=\cos^{-1}(4x^3-3x)$ and $\lim\limits_{x\to\frac{1}{2}^+} f'(x)=a$ and $\lim\limits_{x\to\frac{1}{2}^-} f'(x)=b$, then $a+b+3=$	**(S)**	−2
		(T)	Non existent

Answers

1. A → P ; B → Q ; C → R ; D → S

2. A → Q ; B → P ; C → S ; D → Q

Exercise-5 : Subjective Type Problems

1. If $\lim_{x\to 0} \frac{\ln \cot\left(\frac{\pi}{4}-\beta x\right)}{\tan \alpha x} = 1$, then $\frac{\alpha}{\beta} = ...$

2. If $\lim_{x\to 0} \frac{f(x)}{\sin^2 x} = 8$, $\lim_{x\to 0} \frac{g(x)}{2\cos x - xe^x + x^3 + x - 2} = \lambda$ and $\lim_{x\to 0} (1+2f(x))^{\frac{1}{g(x)}} = \frac{1}{e}$, then $\lambda =$

3. If α, β are two distinct real roots of the equation $ax^3 + x - 1 - a = 0, (a \neq -1, 0)$, none of which is equal to unity, then the value of $\lim_{x\to(1/\alpha)} \frac{(1+a)x^3 - x^2 - a}{(e^{1-\alpha x}-1)(x-1)}$ is $\frac{al(k\alpha - \beta)}{\alpha}$. Find the value of kl.

4. The value of $\lim_{x\to 0} \frac{(140)^x - (35)^x - (28)^x - (20)^x + 7^x + 5^x + 4^x - 1}{x \sin^2 x} = 2\ln 2 \ln k \ln 7$, then $k =$

5. If $\lim_{x\to 0} \frac{a\cot x}{x} + \frac{b}{x^2} = \frac{1}{3}$, then $b - a =$

6. Find the value of $\lim_{x\to\infty} \left(x + \frac{1}{x}\right)e^{1/x} - x$.

7. Find $\lim_{x\to\alpha^+} \left[\frac{\min(\sin x, \{x\})}{x-1}\right]$ where α is root of equation $\sin x + 1 = x$ (here [·] represent greatest integer and {·} represent fractional part function)

Answers

1.	2	**2.**	8	**3.**	1	**4.**	5	**5.**	2	**6.**	1	**7.**	0

❑❑❑

Chapter 3 – Continuity, Differentiability and Differentiation

3 CONTINUITY, DIFFERENTIABILITY AND DIFFERENTIATION

Exercise-1 : Single Choice Problems

1. Let 'f' be a differentiable real valued function satisfying $f(x+2y)=f(x)+f(2y)+6xy(x+2y)\ \forall\ x, y \in R$. Then $f''(0), f''(1), f''(2)......$ are in :

(a) AP (b) GP (c) HP (d) None of these

2. The number of points of non-differentiability for $f(x)=\max\left\{||x|-1|, \frac{1}{2}\right\}$ is :

(a) 4 (b) 3 (c) 2 (d) 5

3. Number of points of discontinuity of $f(x)=\left\{\frac{x}{5}\right\}+\left[\frac{x}{2}\right]$ in $x \in [0, 100]$ is/are (where $[\cdot]$ denotes greatest integer function and $\{\cdot\}$ denotes fractional part function)

(a) 50 (b) 51 (c) 52 (d) 61

4. If $f(x)$ has isolated point of discontinuity at $x=a$ such that $|f(x)|$ is continuous at $x=a$ then :

(a) $\lim_{x\to a} f(x)$ does not exist (b) $\lim_{x\to a} f(x)+f(a)=0$

(c) $f(a)=0$ (d) None of these

5. If $f(x)$ is a thrice differentiable function such that, $\lim_{x\to 0}\frac{f(4x)-3f(3x)+3f(2x)-f(x)}{x^3}=12$ then the value of $f'''(0)$ equals to :

(a) 0 (b) 1 (c) 12 (d) None of these

6. $$y=\frac{1}{1+(\tan\theta)^{\sin\theta-\cos\theta}+(\cot\theta)^{\cos\theta-\cot\theta}}+\frac{1}{1+(\tan\theta)^{\cos\theta-\sin\theta}+(\cot\theta)^{\sin\theta-\cot\theta}}+\frac{1}{1+(\tan\theta)^{\cos\theta-\cot\theta}+(\cot\theta)^{\cot\theta-\sin\theta}}$$ then $\frac{dy}{dx}$ at $\theta=\pi/3$ is :

(a) 0 (b) 1

(c) $\sqrt{3}$ (d) None of these

7. Let $f'(x)=\sin(x^2)$ and $y=f(x^2+1)$ then $\frac{dy}{dx}$ at $x=1$ is :

(a) $2\sin 2$ (b) $2\cos 2$ (c) $2\sin 4$ (d) $\cos 2$

8. If $f(x) = |\sin x - |\cos x||$, then $f'\left(\frac{7\pi}{6}\right) =$

(a) $\frac{\sqrt{3}+1}{2}$ (b) $\frac{1-\sqrt{3}}{2}$

(c) $\frac{\sqrt{3}-1}{2}$ (d) $\frac{-1-\sqrt{3}}{2}$

9. If $2\sin x \cdot \cos y = 1$, then $\frac{d^2y}{dx^2}$ at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ is ....

(a) -4 (b) -2 (c) -6 (d) 0

10. f is a differentiable function such that $x = f(t^2), y = f(t^3)$ and $f'(1) \neq 0$ if $\left(\frac{d^2y}{dx^2}\right)_{t=1} =$

(a) $\frac{3}{4}\left(\frac{f''(1) + f'(1)}{(f'(1))^2}\right)$ (b) $\frac{3}{4}\left(\frac{f'(1) \cdot f''(1) - f''(1)}{(f'(1))^2}\right)$

(c) $\frac{4}{3}\frac{f''(1)}{(f'(1))^2}$ (d) $\frac{4}{3}\left(\frac{f'(1) f''(1) - f''(1)}{(f'(1))^2}\right)$

11. Let $f(x) = \begin{cases} ax+1 & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ bx^2+1 & \text{if } x > 1 \end{cases}$. If $f(x)$ is continuous at $x = 1$ then $(a - b)$ is equal to :

(a) 0 (b) 1 (c) 2 (d) 4

12. If $y = 1 + \frac{\alpha}{\left(\frac{1}{x} - \alpha\right)} + \frac{\beta/x}{\left(\frac{1}{x} - \alpha\right)\left(\frac{1}{x} - \beta\right)} + \frac{\gamma/x^2}{\left(\frac{1}{x} - \alpha\right)\left(\frac{1}{x} - \beta\right)\left(\frac{1}{x} - \gamma\right)}$, then $\frac{dy}{dx}$ is :

(a) $y\left(\frac{\alpha}{\alpha - x} + \frac{\beta}{\beta - x} + \frac{\gamma}{\gamma - x}\right)$ (b) $\frac{y}{x}\left(\frac{\alpha}{1/x - \alpha} + \frac{\beta}{1/x - \beta} + \frac{\gamma}{1/x - \gamma}\right)$

(c) $y\left(\frac{\alpha}{1/x - \alpha} + \frac{\beta}{1/x - \beta} + \frac{\gamma}{1/x - \gamma}\right)$ (d) $\frac{y}{x}\left(\frac{\alpha/x}{1/x - \alpha} + \frac{\beta/x}{1/x - \beta} + \frac{\gamma/x}{1/x - \gamma}\right)$

13. If $f(x) = \sqrt{\frac{1 + \sin^{-1} x}{1 - \tan^{-1} x}}$; then $f'(0)$ is equal to :

(a) 4 (b) 3 (c) 2 (d) 1

14. Let $f(x) = \begin{cases} \sin^2 x & , \ x \text{ is rational} \\ -\sin^2 x & , \ x \text{ is irrational} \end{cases}$, then set of points, where $f(x)$ is continuous, is :

(a) $\left\{(2n+1)\frac{\pi}{2}, n \in I\right\}$ (b) a null set

(c) $\{n\pi, n \in I\}$ (d) set of all rational numbers

15. The number of values of x in $(0, 2\pi)$ where the function $f(x) = \frac{\tan x + \cot x}{2} - \left|\frac{\tan x - \cot x}{2}\right|$ is continuous but non-derivable :

(a) 3 (b) 4 (c) 0 (d) 1

16. If $f(x) = |x-1|$ and $g(x) = f(f(f(x)))$, then $g'(x)$ is equal to :

(a) 1 for $x > 2$ (b) 1 for $2 < x < 3$ (c) -1 for $2 < x < 3$ (d) -1 for $x > 3$

17. If $f(x)$ is a continuous function $\forall\, x \in R$ and the range of $f(x)$ is $(2, \sqrt{26})$ and $g(x) = \left[\frac{f(x)}{C}\right]$ is continuous $\forall\, x \in R$, then the least positive integral value of C is : (where $[\cdot]$ denotes the greatest integer function.)

(a) 3 (b) 5 (c) 6 (d) 7

18. If $y = x + e^x$, then $\left(\frac{d^2x}{dy^2}\right)_{x=\ln 2}$ is :

(a) $-\frac{1}{9}$ (b) $-\frac{2}{27}$ (c) $\frac{2}{27}$ (d) $\frac{1}{9}$

19. Let $f(x) = x^3 + 4x^2 + 6x$ and $g(x)$ be its inverse then the value of $g'(-4)$:

(a) -2 (b) 2 (c) $\frac{1}{2}$ (d) None of these

20. If $f(x) = 2 + |x| - |x-1| - |x+1|$, then $f'\left(-\frac{1}{2}\right) + f'\left(\frac{1}{2}\right) + f'\left(\frac{3}{2}\right) + f'\left(\frac{5}{2}\right)$ is equal to :

(a) 1 (b) -1 (c) 2 (d) -2

21. If $f(x) = \cos(x^2 - 4[x]); 0 < x < 1$, (where $[\cdot]$ denotes greatest integer function) then $f'\left(\frac{\sqrt{\pi}}{2}\right)$ is equal to :

(a) $-\sqrt{\frac{\pi}{2}}$ (b) $\sqrt{\frac{\pi}{2}}$ (c) 0 (d) $\sqrt{\frac{\pi}{4}}$

22. Let $g(x)$ be the inverse of $f(x)$ such that $f'(x) = \frac{1}{1+x^5}$, then $\frac{d^2(g(x))}{dx^2}$ is equal to :

(a) $\frac{1}{1+(g(x))^5}$ (b) $\frac{g'(x)}{1+(g(x))^5}$

(c) $5(g(x))^4(1+(g(x))^5)$ (d) $1+(g(x))^5$

23. Let $f(x) = \begin{cases} \min(x, x^2) & x \geq 0 \\ \max(2x, x-1) & x < 0 \end{cases}$, then which of the following is not true ?

(a) $f(x)$ is not differentiable at $x = 0$

(b) $f(x)$ is not differentiable at exactly two points

(c) $f(x)$ is continuous everywhere

(d) $f(x)$ is strictly increasing $\forall\, x \in R$

24. If $f(x) = \lim_{n\to\infty}\left(\prod_{i=1}^{n}\cos\left(\frac{x}{2^i}\right)\right)$ then $f'(x)$ is equal to :

(a) $\frac{\sin x}{x}$ (b) $\frac{x}{\sin x}$ (c) $\frac{x\cos x - \sin x}{x^2}$ (d) $\frac{\sin x - x\cos x}{\sin^2 x}$

25. Let $f(x) = \begin{cases} \frac{1-\tan x}{4x-\pi} & x \neq \frac{\pi}{4} \\ \lambda & x = \frac{\pi}{4} \end{cases}; x \in \left[0, \frac{\pi}{2}\right)$.

If $f(x)$ is continuous in $\left[0, \frac{\pi}{2}\right)$ then λ is equal to :

(a) 1 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) -1

26. Let $f(x) = \begin{cases} e^{-\frac{1}{x^2}}\sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$, then $f'(0) =$

(a) 1 (b) -1 (c) 0 (d) Does not exist

27. Let f be a differentiable function satisfying $f'(x) = 2f(x) + 10\ \forall\, x \in R$ and $f(0) = 0$, then the number of real roots of the equation $f(x) + 5\sec^2 x = 0$ in $(0, 2\pi)$ is :

(a) 0 (b) 1 (c) 2 (d) 3

28. If $f(x) = \begin{cases} \frac{\sin\{\cos x\}}{x - \frac{\pi}{2}} & x \neq \frac{\pi}{2} \\ 1 & x = \frac{\pi}{2} \end{cases}$, where $\{k\}$ represents the fractional part of k, then :

(a) $f(x)$ is continuous at $x = \frac{\pi}{2}$

(b) $\lim_{x\to\frac{\pi}{2}} f(x)$ does not exist

(c) $\lim_{x\to\frac{\pi}{2}} f(x)$ exists, but f is not continuous at $x = \frac{\pi}{2}$

(d) $\lim_{x\to\frac{\pi}{2}^-} f(x) = 1$

29. Let $f(x)$ be a polynomial in x. The second derivative of $f(e^x)$ w.r.t. x is :

(a) $f''(e^x)e^x + f'(e^x)$ (b) $f''(e^x)e^{2x} + f'(e^x)e^{2x}$

(c) $f''(e^x)e^x + f'(e^x)e^{2x}$ (d) $f''(e^x)e^{2x} + e^x f'(e^x)$

30. If $e^{f(x)} = \log_e x$ and $g(x)$ is the inverse function of $f(x)$, then $g'(x)$ is equal to :

(a) $e^x + x$ (b) $e^{e^{e^x}} e^{e^x} e^x$ (c) $e^{e^x + x}$ (d) e^{e^x}

31. If $y = f(x)$ is differentiable $\forall\, x \in R$, then

(a) $y = |f(x)|$ is differentiable $\forall\, x \in R$

(b) $y = f^2(x)$ is non-differentiable for atleast one x

(c) $y = f(x)\,|f(x)|$ is non-differentiable for atleast one x

(d) $y = |f(x)|^3$ is differentiable $\forall\, x \in R$

32. If $f(x) = (x-1)^4 (x-2)^3 (x-3)^2$ then the value of $f'''(1) + f''(2) + f'(3)$ is :

(a) 0 (b) 1 (c) 2 (d) 6

33. If $f(x) = \left(\frac{x}{2}\right) - 1$, then on the interval $[0, \pi]$:

(a) $\tan(f(x))$ and $\frac{1}{f(x)}$ are both continuous

(b) $\tan(f(x))$ and $\frac{1}{f(x)}$ are both discontinuous

(c) $\tan(f(x))$ and $f^{-1}(x)$ are both continuous

(d) $\tan f(x)$ is continuous but $f^{-1}(x)$ is not

34. Let $f(x) = \begin{cases} \dfrac{e^{\frac{1}{x-2}} - 3}{3^{\frac{1}{x-2}} + 1} & x > 2 \\ \dfrac{b \sin\{-x\}}{\{-x\}} & x < 2 \\ c & x = 2 \end{cases}$, where $\{\cdot\}$ denotes fraction part function, is continuous at $x = 2$,

then $b + c =$

(a) 0 (b) 1 (c) 2 (d) 4

35. Let $f(x) = \dfrac{e^{\tan x} - e^x + \ln(\sec x + \tan x) - x}{\tan x - x}$ be a continuous function at $x = 0$. The value of $f(0)$ equals :

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) 2

36. Let $f(x)=\begin{cases}(1+ax)^{1/x} & x<0\\ b & x=0\\ \dfrac{(x+c)^{1/3}-1}{(x+1)^{1/2}-1} & x>0\end{cases}$, is continuous at $x=0$, then $3(e^a+b+c)$ is equal to :

(a) 3 (b) 6 (c) 7 (d) 8

37. If $\sqrt{x+y}+\sqrt{y-x}=5$, then $\dfrac{d^2y}{dx^2}=$

(a) $\dfrac{2}{5}$ (b) $\dfrac{4}{25}$ (c) $\dfrac{2}{25}$ (d) $\dfrac{1}{25}$

38. If $f(x)=x^3+x^4+\log x$ and g is the inverse of f, then $g'(2)$ is :

(a) 8 (b) $\dfrac{1}{8}$ (c) 2 (d) $\dfrac{1}{4}$

39. The number of points at which the function,

$$f(x)=\begin{cases}\min\{|x|,x^2\} & \text{if } x\in(-\infty,1)\\ \min\{2x-1,x^2\} & \text{otherwise}\end{cases}$$

is not differentiable is :

(a) 0 (b) 1 (c) 2 (d) 3

40. If $f(x)$ is a function such that $f(x)+f''(x)=0$ and $g(x)=(f(x))^2+(f'(x))^2$ and $g(3)=8$, then $g(8)=$

(a) 0 (b) 3 (c) 5 (d) 8

41. Let f is twice differentiable on R such that $f(0)=1, f'(0)=0$ and $f''(0)=-1$, then for $a\in R$,

$$\lim_{x\to\infty}\left(f\left(\frac{a}{\sqrt{x}}\right)\right)^x=$$

(a) e^{-a^2} (b) $e^{-\frac{a^2}{4}}$ (c) $e^{-\frac{a^2}{2}}$ (d) e^{-2a^2}

42. Let $f_1(x)=e^x$ and $f_{n+1}(x)=e^{f_n(x)}$ for any $n\geq 1, n\in N$. Then for any fixed n, the value of $\dfrac{d}{dx}f_n(x)$ equals :

(a) $f_n(x)$ (b) $f_n(x)f_{n-1}(x)\ldots\ldots f_2(x)f_1(x)$

(c) $f_n(x)f_{n-1}(x)$ (d) $f_n(x)f_{n-1}(x)\ldots\ldots f_2(x)f_1(x)e^x$

43. If $y=\tan^{-1}\left(\dfrac{x^{1/3}-a^{1/3}}{1+x^{1/3}a^{1/3}}\right)$, $x>0, a>0$, then $\dfrac{dy}{dx}$ is :

(a) $\dfrac{1}{x^{2/3}(1+x^{2/3})}$ (b) $\dfrac{3}{x^{2/3}(1+x^{2/3})}$ (c) $\dfrac{1}{3x^{2/3}(1+x^{2/3})}$ (d) $\dfrac{1}{3x^{1/3}(1+x^{2/3})}$

44. The value of $k + f(0)$ so that $f(x) = \begin{cases} \dfrac{\sin(4k-1)x}{3x}, & x < 0 \\ \dfrac{\tan(4k+1)x}{5x}, & 0 < x < \dfrac{\pi}{2} \\ 1, & x = 0 \end{cases}$ can be made continuous at $x = 0$ is :

(a) 1 (b) 2 (c) $\dfrac{5}{4}$ (d) 0

45. If $y = \tan^{-1}\left(\dfrac{x}{1+\sqrt{1-x^2}}\right)$, $|x| \le 1$, then $\dfrac{dy}{dx}$ at $\left(\dfrac{1}{2}\right)$ is :

(a) $\dfrac{1}{\sqrt{3}}$ (b) 3 (c) $\dfrac{\sqrt{3}}{2}$ (d) $\dfrac{2}{\sqrt{3}}$

46. Let $f(x) = \dfrac{e^x x\cos x - x\log_e(1+x) - x}{x^2}$, $x \ne 0$. If $f(x)$ is continuous at $x = 0$, then $f(0)$ is equal to :

(a) 0 (b) 1 (c) -1 (d) 2

47. A function $f(x) = \max(\sin x, \cos x, 1-\cos x)$ is non-derivable for n values of $x \in [0, 2\pi]$. Then the value of n is :

(a) 2 (b) 1 (c) 3 (d) 4

48. Let g be the inverse function of a differentiable function f and $G(x) = \dfrac{1}{g(x)}$. If $f(4) = 2$ and $f'(4) = \dfrac{1}{16}$, then the value of $(G'(2))^2$ equals to :

(a) 1 (b) 4 (c) 16 (d) 64

49. If $f(x) = \max.\left(x^4, x^2, \dfrac{1}{81}\right)$ $\forall\, x \in [0, \infty)$, then the sum of the square of reciprocal of all the values of x where $f(x)$ is non-differentiable, is equal to :

(a) 1 (b) 81 (c) 82 (d) $\dfrac{82}{81}$

50. If $f(x)$ is derivable at $x = 2$ such that $f(2) = 2$ and $f'(2) = 4$, then the value of

$$\lim_{h\to 0} \frac{1}{h^2}(\ln(f(2+h^2)) - \ln(f(2-h^2)))$$ is equal to :

(a) 1 (b) 2 (c) 3 (d) 4

51. Let $f(x) = (x^2 - 3x + 2)\,|(x^3 - 6x^2 + 11x - 6)| + \left|\sin\left(x + \dfrac{\pi}{4}\right)\right|$.

Number of points at which the function $f(x)$ is non-differentiable in $[0, 2\pi]$, is :

(a) 5 (b) 4 (c) 3 (d) 2

52. Let f and g be differentiable functions on R (the set of all real numbers) such that $g(1) = 2 = g'(1)$ and $f'(0) = 4$. If $h(x) = f(2xg(x) + \cos \pi x - 3)$ then $h'(1)$ is equal to :

(a) 28 (b) 24 (c) 32 (d) 18

53. If $f(x) = \dfrac{(x+1)^7 \sqrt{1+x^2}}{(x^2-x+1)^6}$, then the value of $f'(0)$ is equal to :

(a) 10 (b) 11 (c) 13 (d) 15

54. Statement-1 : The function $f(x) = \lim\limits_{n\to\infty} \dfrac{\log_e(1+x) - x^{2n}\sin(2x)}{1+x^{2n}}$ is discontinuous at $x = 1$.

Statement-2 : L. H. L = R. H. L $\neq f(1)$.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1

(b) Statement-1 is true, Statement-2 is true and Statement-2 is not the correct explanation for Statement-1

(c) Statement-1 is true, Statement-2 is false

(d) Statement-1 is false, Statement-2 is true

55. If $f(x) = \begin{cases} x & ; \text{ if } x \text{ is rational} \\ 1-x & ; \text{ if } x \text{ is irrational} \end{cases}$, then number of points for $x \in R$, where $y = f(f(x))$ is discontinuous is :

(a) 0 (b) 1 (c) 2 (d) Infinitely many

56. Number of points where $f(x) = \begin{cases} \max(|x^2 - x - 2|, x^2 - 3x) & ; x \geq 0 \\ \max(\ln(-x), e^x) & ; x < 0 \end{cases}$

is non-differentiable will be :

(a) 1 (b) 2 (c) 3 (d) None of these

57. If the function $f(x) = -4e^{\frac{1-x}{2}} + 1 + x + \dfrac{x^2}{2} + \dfrac{x^3}{3}$ and $g(x) = f^{-1}(x)$, then the value of $g'\left(\dfrac{-7}{6}\right)$ equals to :

(a) $\dfrac{1}{5}$ (b) $-\dfrac{1}{5}$ (c) $\dfrac{6}{7}$ (d) $-\dfrac{6}{7}$

58. Find k; if possible ; so that

$$f(x) = \begin{cases} \dfrac{\ln(2-\cos 2x)}{\ln^2(1+\sin 3x)} & ; x < 0 \\ k & ; x = 0 \\ \dfrac{e^{\sin 2x} - 1}{\ln(1+\tan 9x)} & ; x > 0 \end{cases}$$

is continuous at $x = 0$.

(a) $\dfrac{2}{3}$ (b) $\dfrac{1}{9}$ (c) $\dfrac{2}{9}$ (d) Not possible

59. Let $x = \frac{1+t}{t^3}; y = \frac{3}{2t^2} + \frac{2}{t}$ then the value of $\frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^3$ is :

(a) 2 (b) 0 (c) −1 (d) −2

60. If $y^{-2} = 1 + 2\sqrt{2}\cos 2x$, then :

$$\frac{d^2y}{dx^2} = y\,(py^2 + 1)(qy^2 - 1) \text{ then the value of } (p+q) \text{ equals to :}$$

(a) 7 (b) 8 (c) 9 (d) 10

61. Let $f : R \to R$ is not identically zero, differentiable function and satisfy the equations $f(xy) = f(x)\,f(y)$ and $f(x + z) = f(x) + f(z)$, then $f(5) =$

(a) 3 (b) 5 (c) 10 (d) 15

62. Number of points at which the function $f(x) = \begin{cases} \min.(x, x^2) & \text{if } -\infty < x < 1 \\ \min.(2x-1, x^2) & \text{if } x \geq 1 \end{cases}$ is not derivable is :

(a) 0 (b) 1 (c) 2 (d) 3

63. If $y = (x + \sqrt{1 + x^2})^n$, then $(1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is :

(a) n^2y (b) $-n^2y$ (c) $-y$ (d) $2x^2y$

64. If $g(x) = f\left(x - \sqrt{1 - x^2}\right)$ and $f'(x) = 1 - x^2$ then $g'(x)$ equals to :

(a) $1 - x^2$ (b) $\sqrt{1 - x^2}$ (c) $2x\left(x + \sqrt{1 - x^2}\right)$ (d) $2x\left(x - \sqrt{1 - x^2}\right)$

65. Let $f(x) = \lim_{n\to\infty} \frac{\log_e(2 + x) - x^{2n}\sin x}{1 + x^{2n}}$ then :

(a) $f(x)$ is continuous at $x = 1$ (b) $\lim_{x\to 1^-} f(x) = \log_e 3$

(c) $\lim_{x\to 1^+} f(x) = -\sin 1$ (d) $\lim_{x\to 1^+} f(x)$ does not exist

66. Let $f(x + y) = f(x)f(y)$ for all x and y, and $f(5) = -2$, $f'(0) = 3$, then $f'(5)$ is equal to :

(a) 3 (b) 1 (c) −6 (d) 6

67. Let $f(x) = \lim_{n\to\infty} \frac{\log_e(2 + x) - x^{2n}\sin x}{1 + x^{2n}}$ then :

(a) $f(x)$ is continuous at $x = 1$ (b) $\lim_{x\to 1^+} f(x) = \log_e 3$

(c) $\lim_{x\to 1^+} f(x) = -\sin 1$ (d) $\lim_{x\to 1^-} f(x)$ does not exist

68. If $f(x)=\begin{cases}\dfrac{x-e^x+1-\{1-\cos 2x\}}{x^2} & x\neq 0\\ k & x=0\end{cases}$ is continuous at $x=0$ then, which of the following statement is false ?

(a) $k=\frac{-5}{2}$ (b) $\{k\}=\frac{1}{2}$ (c) $[k]=-2$ (d) $[k]\{k\}=\frac{-3}{2}$

(where [·] denotes greatest integer function and {·} denotes fraction part function.)

69. Let $f(x)=||x^2-10x+21|-p|$; then the exhaustive set of values of p for which $f(x)$ has exactly 6 points of non-derivability; is :

(a) $(4,\infty)$ (b) $(0, 4)$ (c) $[0, 4]$ (d) $(-4, 4)$

70. If $f(x)=\sqrt{\dfrac{1+\sin^{-1}x}{1-\tan^{-1}x}}$; then $f'(0)$ is equal to :

(a) 4 (b) 3 (c) 2 (d) 1

71. For $t\in(0,1)$; let $x=\sqrt{2^{\sin^{-1}t}}$ and $y=\sqrt{2^{\cos^{-1}t}}$,

then $1+\left(\dfrac{dy}{dx}\right)^2$ equals :

(a) $\dfrac{x^2}{y^2}$ (b) $\dfrac{y^2}{x^2}$ (c) $\dfrac{x^2+y^2}{y^2}$ (d) $\dfrac{x^2+y^2}{x^2}$

72. Let $f(x)=-1+|x-2|$ and $g(x)=1-|x|$ then set of all possible value(s) of x for which (fog) (x) is discontinuous is :

(a) {0, 1, 2} (b) {0, 2} (c) {0} (d) an empty set

73. If $f(x)=[x]\tan(\pi x)$ then $f'(K^+)$ is equal to ($k\in I$ and [.] denotes greatest integer function):

(a) $(k-1)\pi(-1)^k$ (b) $k\pi$ (c) $k\pi(-1)^{k+1}$ (d) $(k-1)\pi(-1)^{k+1}$

74. If $f(x)=\begin{cases}\dfrac{ae^{\sin x}+be^{-\sin x}-c}{x^2}; & x\neq 0\\ 2\ ; & x=0\end{cases}$ is continuous at $x=0$; then :

(a) $a=b=c$ (b) $a=2b=3c$ (c) $a=b=2c$ (d) $2a=2b=c$

75. If $\tan x\cdot\cot y=\sec\alpha$ where α is constant and $\alpha\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ then $\dfrac{d^2y}{dx^2}$ at $\left(\frac{\pi}{4},\frac{\pi}{4}\right)$ equals to :

(a) 0 (b) 1 (c) 2 (d) 3

76. If $y=(x-3)(x-2)(x-1)\times(x+1)(x+2)(x+3)$, then $\dfrac{d^2y}{dx^2}$ at $x=1$ is :

(a) −101 (b) 48 (c) 56 (d) 190

77. Let $f(x+y)=f(x)f(y)\ \forall x,y\in R, f(0)\neq 0$. If $f(x)$ is continuous at $x=0$, then $f(x)$ is continuous at :

(a) all natural numbers only (b) all integers only

(c) all rational numbers only (d) all real numbers

78. If $f(x)=3x^9-2x^4+2x^3-3x^2+x+\cos x+5$ and $g(x)=f^{-1}(x)$; then the value of $g'(6)$ equals :

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) 3

79. If $y=f(x)$ and $z=g(x)$ then $\frac{d^2y}{dz^2}$ equals

(a) $\frac{g'f''-f'g''}{(g')^2}$ (b) $\frac{g'f''-f'g''}{(g')^3}$ (c) $\frac{f'g''-g'f''}{(g')^3}$ (d) None of these

80. Let $f(x)=\begin{cases} x+1 & ; x<0 \\ |x-1| & ; x\geq 0\end{cases}$ and $g(x)=\begin{cases} x+1 & ; x<0 \\ (x-1)^2 & ; x\geq 0\end{cases}$ then the number of points where $g(f(x))$ is not differentiable.

(a) 0 (b) 1 (c) 2 (d) None of these

81. Let $f(x)=[\sin x]+[\cos x]$, $x\in[0,2\pi]$, where $[\cdot]$ denotes the greatest integer function, total number of points where $f(x)$ is non differentiable is equal to :

(a) 2 (b) 3 (c) 4 (d) 5

82. Let $f(x)=\cos x, g(x)=\begin{cases}\min\{f(t):0\leq t\leq x\} & , x\in[0,\pi] \\ (\sin x)-1 & , x>\pi\end{cases}$

Then

(a) $g(x)$ is discontinuous at $x=\pi$ (b) $g(x)$ is continuous for $x\in[0,\infty)$

(c) $g(x)$ is differentiable at $x=\pi$ (d) $g(x)$ is differentiable for $x\in[0,\infty)$

83. If $f(x)=(4+x)^n, n\in N$ and $f^r(0)$ represents the r^{th} derivative of $f(x)$ at $x=0$, then the value of $\sum_{r=0}^{\infty}\frac{f^r(0)}{r!}$ is equal to :

(a) 2^n (b) 3^n (c) 5^n (d) 4^n

84. Let $f(x)=\begin{cases}\frac{x}{1+|x|} & , |x|\geq 1 \\ \frac{x}{1-|x|} & , |x|<1\end{cases}$, then domain of $f'(x)$ is :

(a) $(-\infty,\infty)$ (b) $(-\infty,\infty)-\{-1,0,1\}$ (c) $(-\infty,\infty)-\{-1,1\}$ (d) $(-\infty,\infty)-\{0\}$

85. If the function $f(x) = -4e^{\frac{1-x}{2}} + 1 + x + \frac{x^2}{2} + \frac{x^3}{3}$ and $g(x) = f^{-1}(x)$, then the value of $g'\left(\frac{-7}{6}\right)$ equals :

(a) $\frac{1}{5}$ (b) $-\frac{1}{5}$ (c) $\frac{6}{7}$ (d) $-\frac{6}{7}$

86. The number of points at which the function $f(x) = (x-|x|)^2(1-x+|x|)^2$ is not differentiable in the interval $(-3, 4)$ is :

(a) Zero (b) One (c) Two (d) Three

87. If $f(x) = \sqrt{\frac{1+\sin^{-1} x}{1-\tan^{-1} x}}$; then $f'(0)$ is equal to :

(a) 4 (b) 3 (c) 2 (d) 1

88. If $f(x) = \begin{cases} e^{x-1} & ; \ 0 \le x \le 1 \\ x+1-\{x\} & ; \ 1 < x < 3 \end{cases}$ and $g(x) = x^2 - ax + b$ such that $f(x)g(x)$ is continuous in $[0, 3)$ then the ordered pair (a, b) is (where $\{\cdot\}$ denotes fractional part function) :

(a) $(2, 3)$ (b) $(1, 2)$ (c) $(3, 2)$ (d) $(2, 2)$

89. Use the following table and the fact that $f(x)$ is invertible and differentiable everywhere to find $f^{-1}(3)$:

x	$f(x)$	$f'(x)$
3	1	7
6	2	10
9	3	5

(a) 0 (b) $\frac{1}{5}$ (c) $\frac{1}{10}$ (d) $\frac{1}{7}$

90. Let $f(x) = \begin{cases} x^n \sin\frac{1}{x} & , \ x \ne 0 \\ 0 & , \ x = 0 \end{cases}$

Such that $f(x)$ is continuous at $x = 0$; $f'(0)$ is real and finite; and $\lim_{x \to 0} f'(x)$ does not exist. This holds true for which of the following values of n ?

(a) 0 (b) 1 (c) 2 (d) 3

Answers

1.	(a)	**2.**	(d)	**3.**	(a)	**4.**	(b)	**5.**	(c)	**6.**	(a)	**7.**	(c)	**8.**	(c)	**9.**	(a)	**10.**	(a)
11.	(a)	**12.**	(b)	**13.**	(d)	**14.**	(c)	**15.**	(b)	**16.**	(c)	**17.**	(c)	**18.**	(b)	**19.**	(c)	**20.**	(d)
21.	(a)	**22.**	(c)	**23.**	(b)	**24.**	(c)	**25.**	(c)	**26.**	(c)	**27.**	(a)	**28.**	(b)	**29.**	(d)	**30.**	(c)
31.	(d)	**32.**	(a)	**33.**	(c)	**34.**	(a)	**35.**	(c)	**36.**	(c)	**37.**	(c)	**38.**	(b)	**39.**	(b)	**40.**	(d)
41.	(c)	**42.**	(b)	**43.**	(c)	**44.**	(b)	**45.**	(a)	**46.**	(a)	**47.**	(c)	**48.**	(a)	**49.**	(c)	**50.**	(d)
51.	(c)	**52.**	(c)	**53.**	(c)	**54.**	(c)	**55.**	(a)	**56.**	(c)	**57.**	(a)	**58.**	(c)	**59.**	(c)	**60.**	(d)
61.	(b)	**62.**	(b)	**63.**	(a)	**64.**	(c)	**65.**	(c)	**66.**	(c)	**67.**	(c)	**68.**	(c)	**69.**	(b)	**70.**	(d)
71.	(d)	**72.**	(d)	**73.**	(b)	**74.**	(d)	**75.**	(a)	**76.**	(c)	**77.**	(d)	**78.**	(a)	**79.**	(b)	**80.**	(c)
81.	(d)	**82.**	(b)	**83.**	(c)	**84.**	(c)	**85.**	(a)	**86.**	(a)	**87.**	(d)	**88.**	(c)	**89.**	(b)	**90.**	(c)

Exercise-2 : One or More than One Answer is/are Correct

1. If $f(x) = \tan^{-1}(\operatorname{sgn}(x^2 - \lambda x + 1))$ has exactly one point of discontinuity, then the value of λ can be :

(a) 1 (b) −1 (c) 2 (d) −2

2. $f(x) = \begin{cases} 2(x+1) & ; \quad x \le -1 \\ \sqrt{1-x^2} & ; \quad -1 < x < 1 \\ \left|\left||x|-1\right|-1\right| & ; \quad x \ge 1 \end{cases}$, then :

(a) $f(x)$ is non-differentiable at exactly three points

(b) $f(x)$ is continuous in $(-\infty, 1]$

(c) $f(x)$ is differentiable in $(-\infty, -1)$

(d) $f(x)$ is finite type of discontinuity at $x = 1$, but continuous at $x = -1$

3. Let $f(x) = \begin{cases} \dfrac{x(3e^{1/x} + 4)}{2 - e^{1/x}} & ; \quad x \ne 0 \quad x \ne \dfrac{1}{\ln 2} \\ 0 & ; \quad x = 0 \end{cases}$

which of the following statement(s) is/are correct ?

(a) $f(x)$ is continuous at $x = 0$

(b) $f(x)$ is non-derivable at $x = 0$

(c) $f'(0^+) = -3$

(d) $f'(0^-)$ does not exist

4. Let $|f(x)| \le \sin^2 x$, $\forall x \in R$, then

(a) $f(x)$ is continuous at $x = 0$

(b) $f(x)$ is differentiable at $x = 0$

(c) $f(x)$ is continuous but not differentiable at $x = 0$

(d) $f(0) = 0$

5. Let $f(x) = \begin{cases} \dfrac{a(1 - x\sin x) + b\cos x + 5}{x^2} & ; \quad x < 0 \\ 3 & ; \quad x = 0 \\ \left(1 + \left(\dfrac{cx + dx^3}{x^2}\right)\right)^{\frac{1}{x}} & ; \quad x > 0 \end{cases}$

If f is continuous at $x = 0$ then correct statement(s) is/are :

(a) $a + c = -1$

(b) $b + c = -4$

(c) $a + b = -5$

(d) $c + d =$ an irrational number

6. If $f(x) = \left|\left||x| - 2\right| + p\right|$ have more than 3 points of non-derivability then the value of p can be :

(a) 0

(b) −1

(c) −2

(d) 2

7. Identify the options having correct statement :

(a) $f(x) = \sqrt[3]{x^2|x|} - 1 - |x|$ is no where non-differentiable

(b) $\lim_{x \to \infty} ((x+5)\tan^{-1}(x+1)) - ((x+1)\tan^{-1}(x+1)) = 2\pi$

(c) $f(x) = \sin(\ln(x + \sqrt{x^2+1}))$ is an odd function

(d) $f(x) = \dfrac{4-x^2}{4x-x^3}$ is discontinuous at exactly one point

8. A twice differentiable function $f(x)$ is defined for all real numbers and satisfies the following conditions :

$f(0) = 2; \quad f'(0) = -5 \quad \text{and} \quad f''(0) = 3.$

The function $g(x)$ is defined by $g(x) = e^{ax} + f(x) \ \forall x \in R$, where '$a$' is any constant. If $g'(0) + g''(0) = 0$ then 'a' can be equal to :

(a) 1 (b) −1 (c) 2 (d) −2

9. If $f(x) = |x| \sin x$, then f is :

(a) differentiable everywhere (b) not differentiable at $x = n\pi, n \in I$

(c) not differentiable at $x = 0$ (d) continuous at $x = 0$

10. Let [ ] denotes the greatest integer function and $f(x) = [\tan^2 x]$, then

(a) $\lim_{x \to 0} f(x)$ does not exist (b) $f(x)$ is continuous at $x = 0$

(c) $f(x)$ is not differentiable at $x = 0$ (d) $f'(0) = 0$

11. Let f be a differentiable function satisfying $f'(x) = f'(-x) \ \forall x \in R$. Then

(a) If $f(1) = f(2)$, then $f(-1) = f(-2)$

(b) $\dfrac{1}{2}f(x) + \dfrac{1}{2}f(y) = f\left(\dfrac{1}{2}(x+y)\right)$ for all real values of x, y

(c) Let $f(x)$ be an even function, then $f(x) = 0 \ \forall x \in R$

(d) $f(x) + f(-x) = 2f(0) \ \forall x \in R$

12. Let $f: R \to R$ be a function, such that $|f(x)| \le x^{4n}$, $n \in N \ \forall x \in R$ then $f(x)$ is :

(a) discontinuous at $x = 0$ (b) continuous at $x = 0$

(c) non-differentiable at $x = 0$ (d) differentiable at $x = 0$

13. Let $f(x) = [x]$ and $g(x) = 0$ when x is an integer and $g(x) = x^2$ when x is not an integer ([ ] is the greatest integer function) then :

(a) $\lim_{x \to 1} g(x)$ exists, but $g(x)$ is not continuous at $x = 1$

(b) $\lim_{x \to 1} f(x)$ does not exist

(c) gof is continuous for all x

(d) fog is continuous for all x

14. Let the function f be defined by $f(x) = \begin{cases} p + qx + x^2 & , \quad x < 2 \\ 2px + 3qx^2 & , \quad x \geq 2 \end{cases}$. Then :

(a) $f(x)$ is continuous in R if $3p + 10q = 4$

(b) $f(x)$ is differentiable in R if $p = q = \dfrac{4}{13}$

(c) If $p = -2, q = 1$, then $f(x)$ is continuous in R

(d) $f(x)$ is differentiable in R if $2p + 11q = 4$

15. Let $f(x) = |2x - 9| + |2x| + |2x + 9|$. Which of the following are true ?

(a) $f(x)$ is not differentiable at $x = \dfrac{9}{2}$

(b) $f(x)$ is not differentiable at $x = \dfrac{-9}{2}$

(c) $f(x)$ is not differentiable at $x = 0$

(d) $f(x)$ is differentiable at $x = \dfrac{-9}{2}, 0, \dfrac{9}{2}$

16. Let $f(x) = \max(x, x^2, x^3)$ in $-2 \leq x \leq 2$. Then :

(a) $f(x)$ is continuous in $-2 \leq x \leq 2$

(b) $f(x)$ is not differentiable at $x = 1$

(c) $f(-1) + f\left(\dfrac{3}{2}\right) = \dfrac{35}{8}$

(d) $f'(-1) f'\left(\dfrac{3}{2}\right) = \dfrac{-35}{4}$

17. If $f(x)$ be a differentiable function satisfying $f(y) f\left(\dfrac{x}{y}\right) = f(x) \; \forall \; x, y \in R, y \neq 0$ and $f(1) \neq 0$, $f'(1) = 3$, then :

(a) $\text{sgn}(f(x))$ is non-differentiable at exactly one point

(b) $\lim\limits_{x \to 0} \dfrac{x^2(\cos x - 1)}{f(x)} = 0$

(c) $f(x) = x$ has 3 solutions

(d) $f(f(x)) - f^3(x) = 0$ has infinitely many solutions

18. Let $f(x) = (x^2 - 3x + 2)(x^2 + 3x + 2)$ and α, β, γ satisfy $\alpha < \beta < \gamma$ are the roots of $f'(x) = 0$ then which of the following is/are correct ([·] denotes greatest integer function) ?

(a) $[\alpha] = -2$

(b) $[\beta] = -1$

(c) $[\beta] = 0$

(d) $[\alpha] = 1$

19. Let the function f be defined by $f(x) = \begin{cases} p + qx + x^2, & x < 2 \\ 2px + 3qx^2, & x \geq 2 \end{cases}$. Then :

(a) $f(x)$ is continuous in R if $3p + 10q = 4$

(b) $f(x)$ is differentiable in R if $p = q = \dfrac{4}{13}$

(c) If $p = -2, q = 1$, then $f(x)$ is continuous in R

(d) $f(x)$ is differentiable in R if $2p + 11q = 4$

20. If $y = e^{x\sin(x^3)} + (\tan x)^x$ then $\frac{dy}{dx}$ may be equal to :

(a) $e^{x\sin(x^3)}[3x^3\cos(x^3) + \sin(x^3)] + (\tan x)^x[\ln\tan x + 2x\operatorname{cosec} 2x]$

(b) $e^{x\sin(x^3)}[x^3\cos(x^3) + \sin(x^3)] + (\tan x)^x[\ln\tan x + 2x\operatorname{cosec} 2x]$

(c) $e^{x\sin(x^3)}[x^3\sin(x^3) + \cos(x^3)] + (\tan x)^x[\ln\tan x + 2\operatorname{cosec} 2x]$

(d) $e^{x\sin(x^3)}[3x^3\cos(x^3) + \sin(x^3)] + (\tan x)^x\left[\ln\tan x + \frac{x\sec^2 x}{\tan x}\right]$

21. Let $f(x) = x + (1-x)x^2 + (1-x)(1-x^2)x^3 + \ldots\ldots + (1-x)(1-x^2)\ldots\ldots(1-x^{n-1})x^n; (n \geq 4)$ then :

(a) $f(x) = -\prod_{r=1}^{n}(1-x^r)$

(b) $f(x) = 1 - \prod_{r=1}^{n}(1-x^r)$

(c) $f'(x) = (1-f(x))\left(\sum_{r=1}^{n}\frac{rx^{r-1}}{(1-x^r)}\right)$

(d) $f'(x) = f(x)\left(\sum_{r=1}^{n}\frac{rx^{r-1}}{(1-x^r)}\right)$

22. Let $f(x) = \begin{cases} x^2 + a\ ;\ 0 \leq x < 1 \\ 2x + b\ ;\ 1 \leq x \leq 2 \end{cases}$ and $g(x) = \begin{cases} 3x + b\ ;\ 0 \leq x < 1 \\ x^3\ ;\ 1 \leq x \leq 2 \end{cases}$

If derivative of $f(x)$ w.r.t. $g(x)$ at $x = 1$ exists and is equal to λ, then which of the following is/are correct ?

(a) $a + b = -3$

(b) $a - b = 1$

(c) $\frac{ab}{\lambda} = 3$

(d) $\frac{-b}{\lambda} = 3$

23. If $f(x) = \begin{cases} \frac{\sin[x^2]\pi}{x^2 - 3x + 8} + ax^3 + b\ ;\ 0 \leq x \leq 1 \\ 2\cos\pi x + \tan^{-1} x\ ;\ 1 < x \leq 2 \end{cases}$ is differentiable in [0, 2] then :

([·] denotes greatest integer function)

(a) $a = \frac{1}{3}$

(b) $a = \frac{1}{6}$

(c) $b = \frac{\pi}{4} - \frac{13}{6}$

(d) $b = \frac{\pi}{4} - \frac{7}{3}$

24. If $f(x) = \begin{cases} 1 + x & 0 \leq x \leq 2 \\ 3 - x & 2 < x \leq 3 \end{cases}$, then $f(f(x))$ is not differentiable at :

(a) $x = 1$

(b) $x = 2$

(c) $x = \frac{5}{2}$

(d) $x = 3$

25. Let $f(x) = (x+1)(x+2)(x+3)\ldots\ldots(x+100)$ and $g(x) = f(x)f''(x) - (f'(x))^2$. Let n be the number of real roots of $g(x) = 0$, then :

(a) $n < 2$

(b) $n > 2$

(c) $n < 100$

(d) $n > 100$

26. If $f(x)=\begin{cases}|x|-3 & , \; x<1 \\ |x-2|+a & , \; x\geq 1\end{cases}$, $g(x)=\begin{cases}2-|x| & , \; x<2 \\ \operatorname{sgn}(x)-b & , \; x\geq 2\end{cases}$

If $h(x)=f(x)+g(x)$ is discontinuous at exactly one point, then which of the following are correct ?

(a) $a=-3, b=0$ (b) $a=-3, b=-1$ (c) $a=2, b=1$ (d) $a=0, b=1$

27. Let $f(x)$ be a continuous function in $[-1, 1]$ such that

$$f(x)=\begin{cases}\dfrac{\ln(ax^2+bx+c)}{x^2} & ; -1\leq x<0 \\ 1 & ; \quad x=0 \\ \dfrac{\sin(e^{x^2}-1)}{x^2} & ; \; 0<x\leq 1\end{cases}$$

Then which of the following is/are correct ?

(a) $a+b+c=0$ (b) $b=a+c$ (c) $c=1+b$ (d) $b^2+c^2=1$

28. $f(x)$ is differentiable function satisfying the relationship $f^2(x)+f^2(y)+2(xy-1)=f^2(x+y)$ $\forall\, x, y \in R$

Also $f(x)>0 \;\forall\, x\in R$ and $f(\sqrt{2})=2$. Then which of the following statement(s) is/are correct about $f(x)$?

(a) $[f(3)]=3$ ([·] denotes greatest integer function)

(b) $f(\sqrt{7})=3$

(c) $f(x)$ is even

(d) $f'(0)=0$

29. The function $f(x)=\left[\sqrt{1-\sqrt{1-x^2}}\right]$, (where [·] denotes greatest integer function) :

(a) has domain $[-1, 1]$

(b) is discontinuous at two points in its domain

(c) is discontinuous at $x=0$

(d) is discontinuous at $x=1$

30. A function $f(x)$ satisfies the relation :

$f(x+y)=f(x)+f(y)+xy(x+y)\;\forall\, x, y\in R$. If $f'(0)=-1$, then :

(a) $f(x)$ is a polynomial function

(b) $f(x)$ is an exponential function

(c) $f(x)$ is twice differentiable for all $x\in R$

(d) $f'(3)=8$

31. The points of discontinuities of $f(x) = \left[\frac{6x}{\pi}\right]\cos\left[\frac{3x}{\pi}\right]$ in $\left[\frac{\pi}{6}, \pi\right]$ is/are :

(where [·] denotes greatest integer function)

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) π

32. Let $f(x) = \begin{cases} \frac{x^2}{2} & 0 \le x < 1 \\ 2x^2 - 3x + \frac{3}{2} & 1 \le x \le 2 \end{cases}$, then in [0, 2] :

(a) $f(x), f'(x)$ are continuous

(b) $f'(x)$ is continuous, $f''(x)$ is not continuous

(c) $f''(x)$ is continuous

(d) $f''(x)$ is non differentiable

33. If $x = \phi(t), y = \psi(t)$, then $\frac{d^2y}{dx^2} =$

(a) $\frac{\phi'\psi'' - \psi'\phi''}{(\phi')^2}$ (b) $\frac{\phi'\psi'' - \psi'\phi''}{(\phi')^3}$ (c) $\frac{\psi''}{\phi'} - \frac{\psi'\phi''}{(\phi')^2}$ (d) $\frac{\psi''}{(\phi')^2} - \frac{\psi'\phi''}{(\phi')^3}$

34. $f(x) = [x]$ and $g(x) = \begin{cases} 0 & , \ x \in I \\ x^2 & , \ x \notin I \end{cases}$ where [·] denotes the greatest integer function. Then

(a) gof is continuous for all x

(b) gof is not continuous for all x

(c) fog is continuous everywhere

(d) fog is not continuous everywhere

35. Let $f : R^+ \to R$ defined as $f(x) = e^x + \ln x$ and $g = f^{-1}$ then correct statement(s) is/are :

(a) $g''(e) = \frac{1-e}{(1+e)^3}$ (b) $g''(e) = \frac{e-1}{(1+e)^3}$ (c) $g'(e) = e+1$ (d) $g'(e) = \frac{1}{e+1}$

36. Let $f(x) = \begin{cases} \frac{3x - x^2}{2} & ; \quad x < 2 \\ [x-1] & ; \quad 2 \le x < 3 \\ x^2 - 8x + 17 & ; \quad x \ge 3 \end{cases}$; then which of the following hold(s) good ?

([·] denotes greatest integer function)

(a) $\lim_{x \to 2} f(x) = 1$ (b) $f(x)$ is differentiable at $x = 2$

(c) $f(x)$ is continuous at $x = 2$ (d) $f(x)$ is discontinuus at $x = 3$

Answers

1.	(c, d)	**2.**	(a, c, d)	**3.**	(a, b, c)	**4.**	(a, b, d)	**5.**	(a, b, c, d)	**6.**	(b, c)
7.	(a, b, c)	**8.**	(a, d)	**9.**	(a, d)	**10.**	(b, d)	**11.**	(a, d)	**12.**	(b, d)
13.	(a, b, c)	**14.**	(a, b, c)	**15.**	(a, b, c)	**16.**	(a, b, c)	**17.**	(a, b, c, d)	**18.**	(a, c)
19.	(a, b, c)	**20.**	(a, d)	**21.**	(b, c)	**22.**	(a, b, c, d)	**23.**	(b, c)	**24.**	(a, b)
25.	(a, c)	**26.**	(a, b, c, d)	**27.**	(c, d)	**28.**	(a, b, c, d)	**29.**	(a, b, d)	**30.**	(a, c, d)
31.	(b, c)	**32.**	(a, b, d)	**33.**	(b, d)	**34.**	(a)	**35.**	(a, d)	**36.**	(a, c, d)

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

Let $f(x) = \lim_{n\to\infty} n^2 \tan\left(\ln\left(\sec\frac{x}{n}\right)\right)$ and $g(x) = \min(f(x), \{x\})$

(where $\{\cdot\}$ denotes fractional part function)

1. Left hand derivative of $\phi(x) = e^{\sqrt{2f(x)}}$ at $x = 0$ is :

(a) 0 (b) 1 (c) -1 (d) Does not exist

2. Number of points in $x \in [-1, 2]$ at which $g(x)$ is discontinuous :

(a) 2 (b) 1 (c) 0 (d) 3

Paragraph for Question Nos. 3 to 4

Let $f(x)$ and $g(x)$ be two differentiable functions, defined as :

$$f(x) = x^2 + xg'(1) + g''(2) \quad \text{and} \quad g(x) = f(1)x^2 + xf'(x) + f''(x).$$

3. The value of $f(1) + g(-1)$ is :

(a) 0 (b) 1 (c) 2 (d) 3

4. The number of integers in the domain of the function $F(x) = \sqrt{-\frac{f(x)}{g(x)}} + \sqrt{3-x}$ is :

(a) 0 (b) 1 (c) 2 (d) Infinite

Paragraph for Question Nos. 5 to 6

Define : $f(x) = |x^2 - 4x + 3| \ln x + 2(x-2)^{1/3}, x > 0$

$$h(x) = \begin{cases} x-1 & , \ x \in Q \\ x^2 - x - 2 & , \ x \notin Q \end{cases}$$

5. $f(x)$ is non-differentiable at .... points and the sum of corresponding x value(s) is ....

(a) 3, 6 (b) 2, 3 (c) 2, 4 (d) 2, 5

6. $h(x)$ is discontinuous at $x =$......

(a) $1+\sqrt{2}$ (b) $\tan\frac{3\pi}{8}$ (c) $\tan\frac{7\pi}{8}$ (d) $\sqrt{2}-1$

Paragraph for Question Nos. 7 to 8

Consider a function defined in $[-2, 2]$

$$f(x)=\begin{cases}\{x\} & -2\le x<-1\\ |\operatorname{sgn} x| & -1\le x\le 1\\ \{-x\} & 1<x\le 2\end{cases}$$

where $\{\cdot\}$ denotes the fractional part function.

7. The total number of points of discontinuity of $f(x)$ for $x \in [-2, 2]$ is :

(a) 0 (b) 1 (c) 2 (d) 4

8. The number of points for $x \in [-2, 2]$ where $f(x)$ is non-differentiable is :

(a) 0 (b) 1 (c) 2 (d) 3

Paragraph for Question Nos. 9 to 10

Consider a function $f(x)$ in $[0, 2\pi]$ defined as :

$$f(x)=\begin{cases}[\sin x]+[\cos x] & ; \quad 0\le x\le \pi\\ [\sin x]-[\cos x] & ; \quad \pi<x\le 2\pi\end{cases}$$

where $[\cdot]$ denotes greatest integer function then

9. Number of points where $f(x)$ is non-derivable :

(a) 2 (b) 3 (c) 4 (d) 5

10. $\lim\limits_{x\to\left(\frac{3\pi}{2}\right)^+} f(x)$ equals

(a) 0 (b) 1 (c) -1 (d) 2

Paragraph for Question Nos. 11 to 13

Let $f(x)=\begin{cases}x[x] & 0\le x<2\\ (x-1)[x] & 2\le x\le 3\end{cases}$ where $[x]$ = greatest integer less than or equal to x, then :

11. The number of values of x for $x \in [0, 3]$ where $f(x)$ is discontinuous is :

(a) 0 (b) 1 (c) 2 (d) 3

12. The number of values of x for $x \in [0, 3]$ where $f(x)$ is non-differentiable is :

(a) 0 (b) 1 (c) 2 (d) 3

13. The number of integers in the range of $y = f(x)$ is :

(a) 3 (b) 4 (c) 5 (d) 6

Paragraph for Question Nos. 14 to 16

Let $f: R \to R$ be a continuous and differentiable function such that $f(x+y) = f(x)\cdot f(y)$ $\forall\, x, y, f(x) \neq 0$ and $f(0) = 1$ and $f'(0) = 2$.

Let $g(xy) = g(x)\cdot g(y)\ \forall\, x, y$ and $g'(1) = 2; g(1) \neq 0$

14. Identify the correct option :

(a) $f(2) = e^4$ (b) $f(2) = 2e^2$ (c) $f(1) < 4$ (d) $f(3) > 729$

15. Identify the correct option :

(a) $g(2) = 2$ (b) $g(3) = 3$ (c) $g(3) = 9$ (d) $g(3) = 6$

16. The number of values of x, where $f(x) = g(x)$:

(a) 0 (b) 1 (c) 2 (d) 3

Paragraph for Question Nos. 17 to 18

Let $f(x) = \dfrac{\cos^2 x}{1 + \cos x + \cos^2 x}$ and $g(x) = \lambda \tan x + (1 - \lambda)\sin x - x$, where $\lambda \in R$ and $x \in [0, \pi/2)$.

17. $g'(x)$ equals

(a) $\dfrac{(1-\cos x)(f(x)-\lambda)}{\cos x}$ (b) $\dfrac{(1-\cos x)(\lambda - f(x))}{\cos x}$

(c) $\dfrac{(1-\cos x)(\lambda - f(x))}{f(x)}$ (d) $\dfrac{(1-\cos x)(\lambda - f(x))}{(f(x))^2}$

18. The exhaustive set of values of 'λ' such that $g'(x) \ge 0$ for any $x \in [0, \pi/2)$:

(a) $[1, \infty)$ (b) $[0, \infty)$ (c) $\left[\dfrac{1}{2}, \infty\right)$ (d) $\left[\dfrac{1}{3}, \infty\right)$

Paragraph for Question Nos. 19 to 21

Let $f(x) = \lim\limits_{n \to \infty} \dfrac{x^2 + 2(x+1)^{2n}}{(x+1)^{2n+1} + x^2 + 1}, n \in N$ and

$g(x) = \tan\left(\dfrac{1}{2}\sin^{-1}\left(\dfrac{2f(x)}{1+f^2(x)}\right)\right)$, then

19. The number of points where $g(x)$ is non-differentiable $\forall\, x \in R$ is :

(a) 1 (b) 2 (c) 3 (d) 4

20. $\lim\limits_{x \to -3} \dfrac{(x^2 + 4x + 3)}{\sin(x+3)\, g(x)}$ is equal to :

(a) 1 (b) 2 (c) 4 (d) Non-existent

21. $\lim\limits_{x \to 0^-} \left\{\dfrac{f(x)}{\tan^2 x}\right\} + \left|\lim\limits_{x \to -2^-} f(x)\right| + \lim\limits_{x \to -2^+} (5f(x))$ is equal to

(where $\{\cdot\}$ denotes fraction part function)

(a) 7 (b) 8 (c) 12 (d) Non-existent

Paragraph for Question Nos. 22 to 24

Let f and g be two differentiable functions such that :

$$f(x) = g'(1)\sin x + (g''(2) - 1)x$$

$$g(x) = x^2 - f'\left(\frac{\pi}{2}\right)x + f''\left(-\frac{\pi}{2}\right)$$

22. The number of solution(s) of the equation $f(x) = g(x)$ is/are :

(a) 1 (b) 2 (c) 3 (d) infinite

23. If $\int \frac{g(\cos x)}{f(x) - x} dx = \cos x + ln\ (h(x)) + C$ where C is constant and $h\left(\frac{\pi}{2}\right) = 1$ then $\left|h\left(\frac{2\pi}{3}\right)\right|$ is :

(a) $3\sqrt{2}$ (b) $2\sqrt{3}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

24. If $\phi(x) = f^{-1}(x)$ then $\phi'\left(\frac{\pi}{2} + 1\right)$ equals to :

(a) $\frac{\pi}{2} + 1$ (b) $\frac{\pi}{2}$ (c) 1 (d) 0

Paragraph for Question Nos. 25 to 26

Suppose a function $f(x)$ satisfies the following conditions

$$f(x + y) = \frac{f(x) + f(y)}{1 + f(x)\, f(y)},\ \forall\ x, y \in R \text{ and } f'(0) = 1$$

Also $-1 < f(x) < 1,\ \forall\ x \in R$

25. $f(x)$ increases in the complete interval :

(a) $(-\infty, -1) \cup (-1, 0) \cup (0, 1) \cup (1, \infty)$ (b) $(-\infty, \infty)$

(c) $(-\infty, -1) \cup (-1, 0)$ (d) $(0, 1) \cup (1, \infty)$

26. The value of the limit $\underset{x \to \infty}{lt}\ (f(x))^x$ is :

(a) 0 (b) 1 (c) e (d) e^2

Paragraph for Question Nos. 27 to 28

Let $f(x)$ be a polynomial satisfying $\lim_{x \to \infty} \frac{x^4 f(x)}{x^8 + 1} = 3$

$f(2) = 5, f(3) = 10, f(-1) = 2, f(-6) = 37$

27. The value of $\lim\limits_{x \to -6} \dfrac{f(x) - x^2 - 1}{3(x+6)}$ equals to :

(a) $-\lfloor 6$ (b) $\lfloor 6$ (c) $\dfrac{\lfloor 6}{2}$ (d) $\dfrac{-\lfloor 6}{2}$

28. The number of points of discontinuity of $g(x) = \dfrac{1}{x^2 + 1 - f(x)}$ in $\left[\dfrac{-15}{2}, \dfrac{5}{2}\right]$ equals :

(a) 4 (b) 3 (c) 1 (d) 0

Paragraph for Question Nos. 29 to 30

Consider $f(x) = x^{\ln x}$ and $g(x) = e^2 x$. Let α and β be two values of x satisfying $f(x) = g(x)$ $(\alpha < \beta)$

29. If $\lim\limits_{x \to \beta} \dfrac{f(x) - c\beta}{g(x) - \beta^2} = l$ then the value of $c - l$ equals to :

(a) $4 - e^2$ (b) $e^2 - 4$ (c) $4 - e$ (d) $e - 4$

30. If $h(x) = \dfrac{f(x)}{g(x)}$ then $h'(\alpha)$ equals to :

(a) e (b) $-e$ (c) $3e$ (d) $-3e$

Paragraph for Question Nos. 31 to 32

Let $f_n(x) + f_n(y) = \dfrac{x^n + y^n}{x^n y^n}$ $\forall\, x, y \in R - \{0\}$ where $n \in N$ and

$g(x) = \max\left\{f_2(x), f_3(x), \dfrac{1}{2}\right\}$ $\forall\, x \in R - \{0\}$

31. The minimum value of $\sum\limits_{k=1}^{\infty} f_{2k}(\operatorname{cosec}\theta) + \sum\limits_{k=1}^{\infty} f_{2k}(\sec\theta)$, where $\theta \neq \dfrac{k\pi}{2}$; $k \in I$ is :

(a) 1 (b) 2 (c) $\sqrt{2}$ (d) 4

32. The number of values of x for which $g(x)$ is non-differentiable $(x \in R - \{0\})$:

(a) 3 (b) 4 (c) 5 (d) 1

Answers

1.	(c)	**2.**	(a)	**3.**	(d)	**4.**	(c)	**5.**	(d)	**6.**	(d)	**7.**	(b)	**8.**	(d)	**9.**	(c)	**10.**	(c)
11.	(c)	**12.**	(d)	**13.**	(c)	**14.**	(a)	**15.**	(c)	**16.**	(b)	**17.**	(c)	**18.**	(d)	**19.**	(d)	**20.**	(b)
21.	(a)	**22.**	(b)	**23.**	(b)	**24.**	(c)	**25.**	(b)	**26.**	(b)	**27.**	(d)	**28.**	(b)	**29.**	(b)	**30.**	(d)
31.	(b)	**32.**	(a)																

Exercise-4 : Matching Type Problems

1.

	Column-I		Column-II
(A)	If $\int_0^{\pi} \frac{\log \sin x}{\cos^2 x} dx = -K$ then the value of $\frac{3k}{\pi}$ is greater than	**(P)**	0
(B)	If $e^{x+y} + e^{y-x} = 1$ and $y'' - (y')^2 + K = 0$, then K is equal to	**(Q)**	1
(C)	If $f(x) = x \ln x$ then $2(f^{-1})'(\ln 4)$ is more than	**(R)**	2
(D)	$\lim_{x \to \infty} (x \ln x)^{\frac{1}{x^2+1}}$ is less than	**(S)**	4
		(T)	5

2. Let $f(x) = \begin{cases} [x] & , \; -2 \le x < 0 \\ |x| & , \; 0 \le x \le 2 \end{cases}$

(where [·] denotes the greatest integer function) $g(x) = \sec x$, $x \in R - (2n+1)\frac{\pi}{2}$, $n \in I$

Match the following statements in column I with their values in column II in the interval $\left(-\frac{3\pi}{2}, \frac{3\pi}{2}\right)$.

	Column-I		Column-II
(A)	Abscissa of points where limit of $fog(x)$ exist is/are	**(P)**	-1
(B)	Abscissa of points in domain of $gof(x)$, where limit of $gof(x)$ does not exist is/are	**(Q)**	π
(C)	Abscissa of points of discontinuity of $fog(x)$ is/are	**(R)**	$\frac{5\pi}{6}$
(D)	Abscissa of points of differentiability of $fog(x)$ is/are	**(S)**	$-\pi$
		(T)	0

3. Let a function $f(x) = [x]\{x\} - |x|$ where [·], {·} are greatest integer and fractional part respectively then match the following List-I with List-II.

	Column-I		Column-II
(A)	$f(x)$ is continuous at x equal to	**(P)**	3
(B)	$\frac{4}{3}\left|\int_2^3 f(x)\,dx\right|$ is equal to	**(Q)**	1

(C)	If $g(x) = x - 1$ and if $f(x) = g(x)$ where $x \in (-3, \infty)$, then number of solutions	(R)	4
(D)	If $l = \lim_{x \to 4^+} f(x)$, then $-l$ is equal to	(S)	2

4.

	Column-I		Column-II
(A)	$\lim_{x \to \infty} \left(\frac{x^2 + 2x - 1}{2x^2 - 3x - 2} \right)^{\frac{2x+1}{2x-1}} =$	(P)	$\frac{1}{2}$
(B)	$\lim_{x \to 0} \frac{\log_{\sec x/2} \cos x}{\log_{\sec x} \cos \frac{x}{2}} =$	(Q)	2
(C)	Let $f(x) = \max.(\cos x, x, 2x - 1)$ where $x \geq 0$ then number of points of non-differentiability of $f(x)$ is	(R)	5
(D)	If $f(x) = [2 + 3 \sin x]$, $0 < x < \pi$ then number of points at which the function is discontinuous, is	(S)	16

5. The function
$$f(x) = \begin{cases} ax(x-1) + b & x < 1 \\ = x - 1 & 1 \leq x \leq 3 \\ = px^2 + qx + 2 & x > 3 \end{cases}$$

if (i) $f(x)$ is continuous for all x

(ii) $f'(1)$ does not exist

(iii) $f'(x)$ is continuous at $x = 3$, then

	Column-I		Column-II
(A)	a cannot has value	(P)	1/3
(B)	b has value	(Q)	0
(C)	p has value	(R)	–1
(D)	q has value	(S)	1

Answers

1.	A → P, Q, R ; B → Q ; C → P, Q ; D → R, S, T
2.	A → P, Q, R, S, T ; B → P, T ; C → Q, S ; D → P, R, T
3.	A → Q ; B → S ; C → P ; D → R
4.	A → P ; B → S ; C → Q ; D → R
5.	A → S ; B → Q ; C → P ; D → R

Exercise-5 : Subjective Type Problems

1. Let $f(x)=\begin{cases} ax(x-1)+b & ; \quad x<1 \\ x+2 & ; \quad 1\le x\le 3 \\ px^2+qx+2 & ; \quad x>3 \end{cases}$ is continuous $\forall\, x\in R$ except $x=1$ but $|f(x)|$ is differentiable everywhere and $f'(x)$ is continuous at $x=3$ and $|a+p+b+q|=k$, then $k=$

2. If $y=\sin(8\sin^{-1}x)$ then $(1-x^2)\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}=-ky$, where $k=$

3. If $y^2=4ax$, then $\dfrac{d^2y}{dx^2}=\dfrac{ka^2}{y^3}$, where $k^2=$

4. The number of values of $x, x\in[-2,3]$ where $f(x)=[x^2]\sin(\pi x)$ is discontinuous is (where [·] denotes greatest integer function)

5. If $f(x)$ is continuous and differentiable in $[-3,9]$ and $f'(x)\in[-2,8]\ \forall\, x\in(-3,9)$. Let N be the number of divisors of the greatest possible value of $f(9)-f(-3)$, then find the sum of digits of N.

6. If $f(x)=\begin{cases} \cos x^3 & ; \quad x<0 \\ \sin x^3-|x^3-1| & ; \quad x\ge 0 \end{cases}$

then find the number of points where $g(x)=f(|x|)$ is non-differentiable.

7. Let $f(x)=x^2+ax+3$ and $g(x)=x+b$, where $F(x)=\lim\limits_{n\to\infty}\dfrac{f(x)+(x^2)^n g(x)}{1+(x^2)^n}$. If $F(x)$ is continuous at $x=1$ and $x=-1$ then find the value of (a^2+b^2).

8. Let $f(x)=\begin{cases} 2-x & , \quad -3\le x\le 0 \\ x-2 & , \quad 0<x<4 \end{cases}$

Then $f^{-1}(x)$ is discontinuous at $x=$

9. If $f(x)+2f(1-x)=x^2+2\ \forall\, x\in R$ and $f(x)$ is a differentiable function, then the value of $f'(8)$ is

10. Let $f(x)=signum\,(x)$ and $g(x)=x(x^2-10x+21)$, then the number of points of discontinuity of $f[g(x)]$ is

11. If $\dfrac{d^2}{dx^2}\left(\dfrac{\sin^4x+\sin^2x+1}{\sin^2x+\sin x+1}\right)=a\sin^2x+b\sin x+c$ then the value of $b+c-a$ is

12. If $f(x)=a\cos(\pi x)+b$, $f'\left(\dfrac{1}{2}\right)=\pi$ and $\displaystyle\int_{1/2}^{3/2} f(x)\,dx=\dfrac{2}{\pi}+1$, then find the value of $-\dfrac{12}{\pi}\left(\dfrac{\sin^{-1}a}{3}+\cos^{-1}b\right)$.

13. Let $\alpha(x) = f(x) - f(2x)$ and $\beta(x) = f(x) - f(4x)$

and $\alpha'(1) = 5$ $\alpha'(2) = 7$

then find the value of $\beta'(1) - 10$

14. Let $f(x) = -4 \cdot e^{\frac{1-x}{2}} + \frac{x^3}{3} + \frac{x^2}{2} + x + 1$ and g be inverse function of f and $h(x) = \frac{a + bx^{3/2}}{x^{5/4}}$,

$h'(5) = 0$, then $\frac{a^2}{5b^2 g'\left(\frac{-7}{6}\right)} =$

15. If $y = e^{2\sin^{-1} x}$ then $\left|\frac{(x^2 - 1)y'' + xy'}{y}\right|$ is equal to

16. Let f be a continuous function on $[0, \infty)$ such that $\lim_{x \to \infty}\left(f(x) + \int_0^x f(t)\,dt\right)$ exists. Find $\lim_{x \to \infty} f(x)$.

17. Let $f(x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5}$ and let $g(x) = f^{-1}(x)$. Find $g'''(0)$.

18. If $f(x) = \begin{cases} \cos x^3 & ; \; x < 0 \\ \sin x^3 - |x^3 - 1| & ; \; x \geq 0 \end{cases}$

then find the number of points where $g(x) = f(|x|)$ is non-differentiable.

19. Let $f : R^+ \longrightarrow R$ be a differentiable function satisfying :

$f(xy) = \frac{f(x)}{y} + \frac{f(y)}{x} \; \forall x, y \in R^+$ also $f(1) = 0; f'(1) = 1$

find $\lim_{x \to e}\left[\frac{1}{f(x)}\right]$ (where $[\cdot]$ denotes greatest integer function).

20. For the curve $\sin x + \sin y = 1$ lying in the first quadrant there exists a constant α for which $\lim_{x \to 0} x^{\alpha} \frac{d^2y}{dx^2} = L$ (not zero), then $2\alpha =$

21. Let $f(x) = x\tan^{-1}(x^2) + x^4$. Let $f^k(x)$ denotes k^{th} derivative of $f(x)$ w.r.t. x, $k \in N$. If $f^{2m}(0) \neq 0, m \in N$, then $m =$

22. If $x = \cos\theta$ and $y = \sin^3\theta$, then $\left|\frac{yd^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right|$ at $\theta = \frac{\pi}{2}$ is :

23. The value of $x, x \in (2, \infty)$ where $f(x) = \sqrt{x + \sqrt{8x - 16}} + \sqrt{x - \sqrt{8x - 16}}$ is not differentiable is :

24. The number of non differentiability points of function $f(x) = \min\left([x], \{x\}, \left|x - \frac{3}{2}\right|\right)$ for $x \in (0, 2)$, where $[\cdot]$ and $\{\cdot\}$ denote greatest integer function and fractional part function respectively.

Answers

1.	3	**2.**	64	**3.**	16	**4.**	8	**5.**	3	**6.**	2	**7.**	17
8.	2	**9.**	4	**10.**	3	**11.**	7	**12.**	2	**13.**	9	**14.**	5
15.	4	**16.**	0	**17.**	1	**18.**	2	**19.**	2	**20.**	3	**21.**	2
22.	3	**23.**	4	**24.**	3								

❑❑❑

Chapter 4 – Application of Derivatives

4 APPLICATION OF DERIVATIVES

Exercise-1 : Single Choice Problems

1. The difference between the maximum and minimum value of the function $f(x) = 3\sin^4 x - \cos^6 x$ is :

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) 3 (d) 4

2. A function $y = f(x)$ has a second order derivative $f''(x) = 6(x-1)$. If its graph passes through the point (2, 1) and at that point the tangent to the graph is $y = 3x - 5$, then the function is :

(a) $(x-1)^2$ (b) $(x-1)^3$ (c) $(x+1)^3$ (d) $(x+1)^2$

3. If the subnormal at any point on the curve $y = 3^{1-k} \cdot x^k$ is of constant length then k equals to :

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 0

4. If $x^5 - 5qx + 4r$ is divisible by $(x-c)^2$ then which of the following must hold true $\forall\, q, r, c \in R$?

(a) $q = r$ (b) $q + r = 0$ (c) $q^5 = r^4$ (d) $q^4 = r^5$

5. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of $50\,\text{cm}^3/\text{min}$. When the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases, is :

(a) $\frac{1}{36\pi}$ cm/min (b) $\frac{1}{18\pi}$ cm/min (c) $\frac{1}{54\pi}$ cm/min (d) $\frac{5}{6\pi}$ cm/min

6. If $f(x) = \frac{(x-1)(x-2)}{(x-3)(x-4)}$, then number of local extremas for $g(x)$, where $g(x) = f(|x|)$:

(a) 3 (b) 4 (c) 5 (d) None of these

7. Two straight roads OA and OB intersect at an angle 60°. A car approaches O from A, where $OA = 700$ m at a uniform speed of 20 m/s, Simultaneously, a runner starts running from O towards B at a uniform speed of 5 m/s. The time after start when the car and the runner are closest is :

(a) 10 sec (b) 15 sec

(c) 20 sec (d) 30 sec

8. Let $f(x)=\begin{cases} a-3x & ; \ -2\le x<0 \\ 4x+3 & ; \ 0\le x<1 \end{cases}$; if $f(x)$ has smallest value at $x=0$, then range of a, is :

(a) $(-\infty, 3)$ (b) $(-\infty, 3]$ (c) $(3, \infty)$ (d) $[3, \infty)$

9. $f(x)=\begin{cases} 3+|x-k| & , \ x\le k \\ a^2-2+\dfrac{\sin(x-k)}{(x-k)} & , \ x>k \end{cases}$ has minimum at $x=k$, then :

(a) $a\in R$ (b) $|a|<2$ (c) $|a|>2$ (d) $1<|a|<2$

10. For a certain curve $\dfrac{d^2y}{dx^2}=6x-4$ and curve has local minimum value 5 at $x=1$. Let the global maximum and global minimum values, where $0\le x\le 2$; are M and m. Then the value of $(M-m)$ equals to :

(a) -2 (b) 2 (c) 12 (d) -12

11. The tangent to $y=ax^2+bx+\dfrac{7}{2}$ at (1, 2) is parallel to the normal at the point (–2, 2) on the curve $y=x^2+6x+10$. Then the value of $\dfrac{a}{2}-b$ is :

(a) 2 (b) 0 (c) 3 (d) 1

12. If (a, b) be the point on the curve $9y^2=x^3$ where normal to the curve make equal intercepts with the axis, then the value of $(a+b)$ is :

(a) 0 (b) $\dfrac{10}{3}$ (c) $\dfrac{20}{3}$ (d) None of these

13. The curve $y=f(x)$ satisfies $\dfrac{d^2y}{dx^2}=6x-4$ and $f(x)$ has a local minimum value 5 when $x=1$. Then $f(0)$ is equal to :

(a) 1 (b) 0 (c) 5 (d) None of these

14. Let A be the point where the curve $5\alpha^2x^3+10\alpha x^2+x+2y-4=0$ ($\alpha\in R, \alpha\ne 0$) meets the y-axis, then the equation of tangent to the curve at the point where normal at A meets the curve again, is :

(a) $x-\alpha y+2\alpha=0$ (b) $\alpha x+y-2=0$ (c) $2x-y+2=0$ (d) $x+2y-4=0$

15. The difference between the greatest and the least value of the function $f(x)=\cos x+\dfrac{1}{2}\cos 2x-\dfrac{1}{3}\cos 3x$

(a) $\dfrac{11}{5}$ (b) $\dfrac{13}{6}$ (c) $\dfrac{9}{4}$ (d) $\dfrac{7}{3}$

16. The x co-ordinate of the point on the curve $y=\sqrt{x}$ which is closest to the point (2, 1) is :

(a) $\dfrac{2+\sqrt{3}}{2}$ (b) $\dfrac{1+\sqrt{3}}{2}$ (c) $\dfrac{-1+\sqrt{3}}{2}$ (d) 1

17. The tangent at a point P on the curve $y = \ln\left(\frac{2+\sqrt{4-x^2}}{2-\sqrt{4-x^2}}\right) - \sqrt{4-x^2}$ meets the y-axis at T; then PT^2 equals to :

(a) 2 (b) 4 (c) 8 (d) 16

18. Let $f(x) = \int_{x^2}^{x^3} \frac{dt}{\ln t}$ for $x > 1$

and $g(x) = \int_1^x (2t^2 - \ln t) f(t)\, dt$ $(x > 1)$, then :

(a) g is increasing on $(1, \infty)$
(b) g is decreasing on $(1, \infty)$
(c) g is increasing on $(1, 2)$ and decreasing on $(2, \infty)$
(d) g is decreasing on $(1, 2)$ and increasing on $(2, \infty)$

19. Let $f(x) = x^3 + 6x^2 + ax + 2$, if $(-3, -1)$ is the largest possible interval for which $f(x)$ is decreasing function, then $a =$

(a) 3 (b) 9 (c) -2 (d) 1

20. Let $f(x) = \tan^{-1}\left(\frac{1-x}{1+x}\right)$. Then difference of the greatest and least value of $f(x)$ on $[0, 1]$ is :

(a) $\pi/2$ (b) $\pi/4$ (c) π (d) $\pi/3$

21. The number of integral values of a for which $f(x) = x^3 + (a+2)x^2 + 3ax + 5$ is monotonic in $\forall x \in R$.

(a) 2 (b) 4 (c) 6 (d) 7

22. The number of critical points of $f(x) = \left(\int_0^x (\cos^2 t - \sqrt[3]{t})\, dt\right) + \frac{3}{4}x^{4/3} - \frac{x+1}{2}$ in $[0, 6\pi]$ is :

(a) 10 (b) 8 (c) 6 (d) 12

23. Let $f(x) = \min\left(\frac{1}{2} - \frac{3x^2}{4}, \frac{5x^2}{4}\right)$ for $0 \le x \le 1$, then maximum value of $f(x)$ is :

(a) 0 (b) $\frac{5}{64}$

(c) $\frac{5}{4}$ (d) $\frac{5}{16}$

24. Let $f(x) = \begin{cases} 2 - |x^2 + 5x + 6| & x \ne -2 \\ b^2 + 1 & x = -2 \end{cases}$

Has relative maximum at $x = -2$, then complete set of values b can take is :

(a) $|b| \ge 1$ (b) $|b| < 1$ (c) $b > 1$ (d) $b < 1$

25. Let for the function $f(x) = \begin{cases} \cos^{-1} x & ; -1 \le x \le 0 \\ mx + c & ; 0 < x \le 1 \end{cases}$;

Lagrange's mean value theorem is applicable in [–1, 1] then ordered pair (m, c) is :

(a) $\left(1, -\frac{\pi}{2}\right)$ (b) $\left(1, \frac{\pi}{2}\right)$ (c) $\left(-1, -\frac{\pi}{2}\right)$ (d) $\left(-1, \frac{\pi}{2}\right)$

26. Tangents are drawn to $y = \cos x$ from origin then points of contact of these tangents will always lie on :

(a) $\frac{1}{x^2} = \frac{1}{y^2} + 1$ (b) $\frac{1}{x^2} = \frac{1}{y^2} - 2$ (c) $\frac{1}{y^2} = \frac{1}{x^2} + 1$ (d) $\frac{1}{y^2} = \frac{1}{x^2} - 2$

27. Least natural number a for which $x + ax^{-2} > 2 \; \forall \; x \in (0, \infty)$ is :

(a) 1 (b) 2 (c) 5 (d) None of these

28. Angle between the tangents to the curve $y = x^2 - 5x + 6$ at points (2, 0) and (3, 0) is :

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

29. Difference between the greatest and least values of the function $f(x) = \int_0^x (\cos^2 t + \cos t + 2)\, dt$ in the interval $[0, 2\pi]$ is $K\pi$, then K is equal to :

(a) 1 (b) 3 (c) 5 (d) None of these

30. The range of the function $f(\theta) = \frac{\sin\theta}{\theta} + \frac{\theta}{\tan\theta}, \theta \in \left(0, \frac{\pi}{2}\right)$ is equal to :

(a) $(0, \infty)$ (b) $\left(\frac{1}{\pi}, 2\right)$ (c) $(2, \infty)$ (d) $\left(\frac{2}{\pi}, 2\right)$

31. Number of integers in the range of c so that the equation $x^3 - 3x + c = 0$ has all its roots real and distinct is :

(a) 2 (b) 3 (c) 4 (d) 5

32. Let $f(x) = \int e^x (x-1)(x-2)\, dx$. Then $f(x)$ decreases in the interval :

(a) $(2, \infty)$ (b) $(-2, -1)$

(c) $(1, 2)$ (d) $(-\infty, 1) \cup (2, \infty)$

33. If the cubic polynomial $y = ax^3 + bx^2 + cx + d \; (a, b, c, d \in R)$ has only one critical point in its entire domain and $ac = 2$, then the value of $|b|$ is :

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\sqrt{5}$ (d) $\sqrt{6}$

34. On the curve $y = \frac{1}{1+x^2}$, the point at which $\left|\frac{dy}{dx}\right|$ is greatest in the first quadrant is :

(a) $\left(\frac{1}{2}, \frac{4}{5}\right)$ (b) $\left(1, \frac{1}{2}\right)$ (c) $\left(\frac{1}{\sqrt{2}}, \frac{2}{3}\right)$ (d) $\left(\frac{1}{\sqrt{3}}, \frac{3}{4}\right)$

35. If $f(x) = 2x, g(x) = 3\sin x - x\cos x$, then for $x \in \left(0, \frac{\pi}{2}\right)$:

(a) $f(x) > g(x)$ (b) $f(x) < g(x)$

(c) $f(x) = g(x)$ has exactly one real root. (d) $f(x) = g(x)$ has exactly two real roots

36. Let $f(x) = \sin^{-1}\left(\frac{2g(x)}{1+g(x)^2}\right)$, then which are correct ?

(i) $f(x)$ is decreasing if $g(x)$ is increasing and $|g(x)| > 1$

(ii) $f(x)$ is an increasing function if $g(x)$ is increasing and $|g(x)| \le 1$

(iii) $f(x)$ is decreasing function if $g(x)$ is decreasing and $|g(x)| > 1$

(a) (i) and (iii) (b) (i) and (ii) (c) (i), (ii) and (iii) (d) (iii)

37. The graph of the function $y = f(x)$ has a unique tangent at $(e^a, 0)$ through which the graph passes then $\lim_{x \to e^a} \frac{\ln(1+7f(x)) - \sin(f(x))}{3f(x)}$ is equal to :

(a) 1 (b) 3 (c) 2 (d) 7

38. Let $f(x)$ be a function such that $f'(x) = \log_{1/3}(\log_3(\sin x + a))$. The complete set of values of 'a' for which $f(x)$ is strictly decreasing for all real values of x is :

(a) $[4, \infty)$ (b) $[3, 4]$ (c) $(-\infty, 4)$ (d) $[2, \infty)$

39. If $f(x) = a\ln|x| + bx^2 + x$ has extremas at $x = 1$ and $x = 3$, then :

(a) $a = \frac{3}{4}, b = -\frac{1}{8}$ (b) $a = \frac{3}{4}, b = \frac{1}{8}$ (c) $a = -\frac{3}{4}, b = -\frac{1}{8}$ (d) $a = -\frac{3}{4}, b = \frac{1}{8}$

40. Let $f(x) = \begin{cases} 1+\sin x, & x < 0 \\ x^2 - x + 1, & x \ge 0 \end{cases}$, then :

(a) f has a local maximum at $x = 0$ (b) f has a local minimum at $x = 0$

(c) f is increasing everywhere (d) f is decreasing everywhere

41. If m and n are positive integers and $f(x) = \int_1^x (t-a)^{2n}(t-b)^{2m+1}\,dt,\ a \ne b$, then :

(a) $x = b$ is a point of local minimum (b) $x = b$ is a point of local maximum

(c) $x = a$ is a point of local minimum (d) $x = a$ is a point of local maximum

42. For any real θ, the maximum value of $\cos^2(\cos\theta) + \sin^2(\sin\theta)$ is :

(a) 1 (b) $1 + \sin^2 1$

(c) $1 + \cos^2 1$ (d) Does not exist

43. If the tangent at P of the curve $y^2 = x^3$ intersects the curve again at Q and the straight line OP, OQ have inclinations a, b where O is origin, then $\left(\frac{\tan\alpha}{\tan\beta}\right)$ has the value, equals to :

(a) -1 (b) -2 (c) 2 (d) $\sqrt{2}$

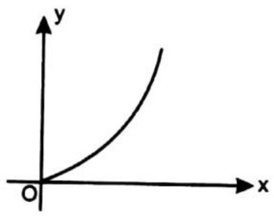
44. If $x + 4y = 14$ is a normal to the curve $y^2 = \alpha x^3 - \beta$ at (2, 3), then value of $\alpha + \beta$ is :

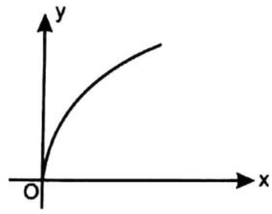
(a) 9 (b) −5 (c) 7 (d) −7

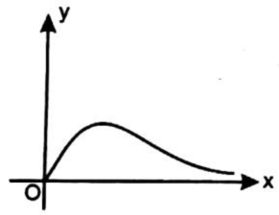
45. The tangent to the curve $y = e^{kx}$ at a point (0, 1) meets the x-axis at $(a, 0)$ where $a \in [-2, -1]$, then $k \in$:

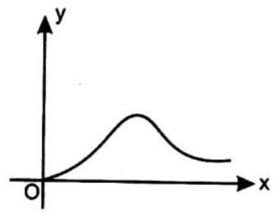
(a) $\left[-\frac{1}{2}, 0\right]$ (b) $\left[-1, -\frac{1}{2}\right]$ (c) $[0, 1]$ (d) $\left[\frac{1}{2}, 1\right]$

46. Which of the following graph represent the function $f(x) = \int_0^{\sqrt{x}} e^{-\frac{u^2}{x}} du$, for $x > 0$ and $f(0) = 0$?

(a)


(b)


(c)


(d)


47. Let $f(x) = (x-a)(x-b)(x-c)$ be a real valued function where $a < b < c$ $(a, b, c \in R)$ such that $f''(\alpha) = 0$. Then if $\alpha \in (c_1, c_2)$, which one of the following is correct ?

(a) $a < c_1 < b$ and $b < c_2 < c$ (b) $a < c_1, c_2 < b$

(c) $b < c_1, c_2 < c$ (d) None of these

48. $f(x) = x^6 - x - 1, x \in [1, 2]$. Consider the following statements :

(1) f is increasing on [1, 2] (2) f has a root in [1, 2]

(3) f is decreasing on [1, 2] (4) f has no root in [1, 2]

Which of the above are correct?

(a) 1 and 2 (b) 1 and 4 (c) 2 and 3 (d) 3 and 4

49. Which one of the following curves is the orthogonal trajectory of straight lines passing through a fixed point (a, b)?

(a) $x - a = k(y - b)$ (b) $(x-a)(y-b) = k$

(c) $(x-a)^2 = k(y-b)$ (d) $(x-a)^2 + (y-b)^2 = k$

50. The function $f(x) = \sin^3 x - m \sin x$ is defined on open interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and if assumes only 1 maximum value and only 1 minimum value on this interval. Then, which one of the following must be correct ?

(a) $0 < m < 3$ (b) $-3 < m < 0$ (c) $m > 3$ (d) $m < -3$

51. The greatest of the numbers 1, $2^{1/2}$, $3^{1/3}$, $4^{1/4}$, $5^{1/5}$, $6^{1/6}$ and $7^{1/7}$ is :

(a) $2^{1/2}$ (b) $3^{1/3}$ (c) $7^{1/7}$ (d) $6^{1/6}$

52. Let l be the line through (0, 0) and tangent to the curve $y = x^3 + x + 16$. Then the slope of l equal to :

(a) 10 (b) 11 (c) 17 (d) 13

53. The slope of the tangent at the point of inflection of $y = x^3 - 3x^2 + 6x + 2009$ is equal to :

(a) 2 (b) 3 (c) 1 (d) 4

54. Let f be a real valued function with $(n + 1)$ derivatives at each point of R. For each pair of real numbers a, b, $a < b$, such that

$$ln\left[\frac{f(b) + f'(b) + \ldots\ldots + f^{(n)}(b)}{f(a) + f'(a) + \ldots\ldots + f^{(n)}(a)}\right] = b - a$$

Statement-1 : There is a number $c \in (a, b)$ for which $f^{(n+1)}(c) = f(c)$

because

Statement-2 : If $h(x)$ be a derivable function such that $h(p) = h(q)$ then by Rolle's theorem $h'(d) = 0$; $d \in (p, q)$

(a) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1

(b) Statement-1 is true, statement-2 is true and statement-2 is not correct explanation for statement-1

(c) Statement-1 is true, statement-2 is false

(d) Statement-1 is false, statement-2 is true

55. If $g(x)$ is twice differentiable real valued function satisfying $g''(x) - 3g'(x) > 3 \ \forall \ x \geq 0$ and $g'(0) = -1$, then $h(x) = g(x) + x \ \forall x > 0$ is :

(a) strictly increasing (b) strictly decreasing

(c) non monotonic (d) data insufficient

56. If the straight line joining the points (0, 3) and (5, –2) is tangent to the curve $y = \frac{c}{x+1}$; then the value of c is :

(a) 2 (b) 3 (c) 4 (d) 5

57. Number of solutions(s) of $\ln|\sin x| = -x^2$ if $x \in \left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$ is/are :

(a) 2 (b) 4 (c) 6 (d) 8

58. The equation $\sin^{-1} x = |x - a|$ will have atleast one solution then complete set of values of a be :

(a) $[-1, 1]$ (b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (c) $\left[1-\frac{\pi}{2}, 1+\frac{\pi}{2}\right]$ (d) $\left[\frac{\pi}{2}-1, \frac{\pi}{2}+1\right]$

59. For any real number b, let $f(b)$ denotes the maximum of $\left|\sin x + \frac{2}{3+\sin x} + b\right| \forall \times x \in R$.

Then the minimum value of $f(b) \forall b \in R$ is :

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) 1

60. Which of the following are correct

(a) $x^4 + 2x^2 - 6x + 2 = 0$ has exactly four real solution

(b) $x^5 + 5x + 1 = 0$ has exactly three real solutions

(c) $x^n + ax + b = 0$ where n is an even natural number has atmost two real solution $a, b, \in R$.

(d) $x^3 - 3x + c = 0, c > 0$ has two real solution for $x \in (0, 1)$

61. For any real number b, let $f(b)$ denotes the maximum of $\left|\sin x + \frac{2}{3+\sin x} + b\right| \forall x \in R$. Then the minimum value of $f(b) \forall b \in R$ is :

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) 1

62. If p be a point on the graph of $y = \frac{x}{1+x^2}$, then coordinates of 'p' such that tangent drawn to curve at p has the greatest slope in magnitude is :

(a) $(0, 0)$ (b) $\left(\sqrt{3}, \frac{\sqrt{3}}{4}\right)$ (c) $\left(-\sqrt{3}, -\frac{\sqrt{3}}{4}\right)$ (d) $\left(1, \frac{1}{2}\right)$

63. Let $f : [0, 2\pi] \to [-3, 3]$ be a given function defined as $f(x) = 3\cos\frac{x}{2}$. The slope of the tangent to the curve $y = f^{-1}(x)$ at the point where the curve crosses the y-axis is :

(a) -1 (b) $-\frac{2}{3}$ (c) $-\frac{1}{6}$ (d) $-\frac{1}{3}$

64. Number of stationary points in $[0, \pi]$ for the function $f(x) = \sin x + \tan x - 2x$ is :

(a) 0 (b) 1 (c) 2 (d) 3

65. If $a, b, c, d \in R$ such that $\frac{a+2c}{b+3d} + \frac{4}{3} = 0$, then the equation $ax^3 + bx^2 + cx + d = 0$ has

(a) atleast one root in $(-1, 0)$ (b) atleast one root in $(0, 1)$

(c) no root in $(-1, 1)$ (d) no root in $(0, 2)$

66. If $f'(x) = \phi(x)(x-2)^2$. Where $\phi(2) \neq 0$ and $\phi(x)$ is continuous at $x = 2$, then in the neighbourhood of $x = 2$

(a) f is increasing if $\phi(2) < 0$
(b) f is decreasing if $\phi(2) > 0$
(c) f is neither increasing nor decreasing
(d) f is increasing if $\phi(2) > 0$

67. If $f(x) = x^3 - 6x^2 + ax + b$ is defined on [1, 3] satisfies Rolle's theorem for $c = \frac{2\sqrt{3}+1}{\sqrt{3}}$ then

(a) $a = -11, b = 6$
(b) $a = -11, b = -6$
(c) $a = 11, b \in R$
(d) $a = 22, b = -6$

68. For which of the following function(s) Lagrange's mean value theorem is not applicable in [1, 2]?

(a) $f(x) = \begin{cases} \frac{3}{2} - x & , x < \frac{3}{2} \\ \left(\frac{3}{2} - x\right)^2 & , x \geq \frac{3}{2} \end{cases}$
(b) $f(x) = \begin{cases} \frac{\sin(x-1)}{x-1} & , x \neq 1 \\ 1 & , x = 1 \end{cases}$
(c) $f(x) = (x-1)|x-1|$
(d) $f(x) = |x-1|$

69. If the curves $\frac{x^2}{a^2} + \frac{y^2}{4} = 1$ and $y^2 = 16x$ intersect at right angles, then :

(a) $a = \pm 1$
(b) $a = \pm\sqrt{3}$
(c) $a = \pm\frac{1}{\sqrt{3}}$
(d) $a = \pm\sqrt{2}$

70. If the line $x\cos\alpha + y\sin\alpha = P$ touches the curve $4x^3 = 27ay^2$, then $\frac{P}{a} =$

(a) $\cot^2\alpha\cos\alpha$
(b) $\cot^2\alpha\sin\alpha$
(c) $\tan^2\alpha\cos\alpha$
(d) $\tan^2\alpha\sin\alpha$

Answers

1.	(d)	**2.**	(b)	**3.**	(a)	**4.**	(c)	**5.**	(b)	**6.**	(c)	**7.**	(d)	**8.**	(d)	**9.**	(c)	**10.**	(b)
11.	(c)	**12.**	(c)	**13.**	(c)	**14.**	(c)	**15.**	(c)	**16.**	(a)	**17.**	(b)	**18.**	(a)	**19.**	(b)	**20.**	(b)
21.	(b)	**22.**	(d)	**23.**	(d)	**24.**	(a)	**25.**	(d)	**26.**	(c)	**27.**	(b)	**28.**	(d)	**29.**	(c)	**30.**	(d)
31.	(b)	**32.**	(c)	**33.**	(d)	**34.**	(d)	**35.**	(a)	**36.**	(b)	**37.**	(c)	**38.**	(a)	**39.**	(c)	**40.**	(a)
41.	(a)	**42.**	(b)	**43.**	(b)	**44.**	(a)	**45.**	(d)	**46.**	(b)	**47.**	(a)	**48.**	(a)	**49.**	(d)	**50.**	(a)
51.	(b)	**52.**	(d)	**53.**	(b)	**54.**	(a)	**55.**	(a)	**56.**	(c)	**57.**	(b)	**58.**	(c)	**59.**	(b)	**60.**	(c)
61.	(b)	**62.**	(a)	**63.**	(b)	**64.**	(c)	**65.**	(b)	**66.**	(d)	**67.**	(c)	**68.**	(a)	**69.**	(d)	**70.**	(a)

Exercise-2 : One or More than One Answer is/are Correct

1. Common tangent(s) to $y = x^3$ and $x = y^3$ is/are :

(a) $x - y = \frac{1}{\sqrt{3}}$ (b) $x - y = -\frac{1}{\sqrt{3}}$ (c) $x - y = \frac{2}{3\sqrt{3}}$ (d) $x - y = \frac{-2}{3\sqrt{3}}$

2. Let $f:[0, 8] \to R$ be differentiable function such that $f(0) = 0, f(4) = 1, f(8) = 1$, then which of the following hold(s) good ?

(a) There exist some $c_1 \in (0, 8)$ where $f'(c_1) = \frac{1}{4}$

(b) There exist some $c \in (0, 8)$ where $f'(c) = \frac{1}{12}$

(c) There exist $c_1, c_2 \in [0, 8]$ where $8f'(c_1)f(c_2) = 1$

(d) There exist some $\alpha, \beta \in (0, 2)$ such that $\int_0^8 f(t)\,dt = 3(\alpha^2 f(\alpha^3) + \beta^2 f(\beta^3))$

3. If $f(x) = \begin{cases} \sin^{-1}(\sin x) & x > 0 \\ \frac{\pi}{2} & x = 0 \\ \cos^{-1}(\cos x) & x < 0 \end{cases}$, then

(a) $x = 0$ is a point of maxima

(b) $f(x)$ is continuous $\forall\, x \in R$

(c) global maximum value of $f(x)\ \forall\, x \in R$ is π

(d) global minimum value of $f(x)\ \forall\, x \in R$ is 0

4. A function $f:R \to R$ is given by $f(x) = \begin{cases} x^4\left(2 + \sin\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$, then

(a) f has a continuous derivative $\forall\, x \in R$ (b) f is a bounded function

(c) f has an global minimum at $x = 0$ (d) f'' is continuous $\forall\, x \in R$

5. If $|f''(x)| \leq 1\ \forall\, x \in R$, and $f(0) = 0 = f'(0)$, then which of the following can not be true ?

(a) $f\left(-\frac{1}{2}\right) = \frac{1}{6}$ (b) $f(2) = -4$ (c) $f(-2) = 3$ (d) $f\left(\frac{1}{2}\right) = \frac{1}{5}$

6. Let $f:[-3, 4] \to R$ such that $f''(x) > 0$ for all $x \in [-3, 4]$, then which of the following are always true ?

(a) $f(x)$ has a relative minimum on $(-3, 4)$

(b) $f(x)$ has a minimum on $[-3, 4]$

(c) $f(x)$ has a maximum on $[-3, 4]$

(d) if $f(3) = f(4)$, then $f(x)$ has a critical point on $[-3, 4]$

7. Let $f(x)$ be twice differentiable function such that $f''(x) > 0$ in $[0, 2]$. Then :

(a) $f(0) + f(2) = 2f(c)$, for atleast one $c, c \in (0, 2)$

(b) $f(0) + f(2) < 2f(1)$

(c) $f(0) + f(2) > 2f(1)$

(d) $2f(0) + f(2) > 3f\left(\frac{2}{3}\right)$

8. Let $g(x)$ be a cubic polynomial having local maximum at $x = -1$ and $g'(x)$ has a local minimum at $x = 1$. If $g(-1) = 10, g(3) = -22$, then :

(a) perpendicular distance between its two horizontal tangents is 12

(b) perpendicular distance between its two horizontal tangents is 32

(c) $g(x) = 0$ has atleast one real root lying in interval $(-1, 0)$

(d) $g(x) = 0$, has 3 distinct real roots

9. The function $f(x) = 2x^3 - 3(\lambda + 2)x^2 + 2\lambda x + 5$ has a maximum and a minimum for :

(a) $\lambda \in (-4, \infty)$ (b) $\lambda \in (-\infty, 0)$ (c) $\lambda \in (-3, 3)$ (d) $\lambda \in (1, \infty)$

10. The function $f(x) = 1 + x\ln\left(x + \sqrt{1 + x^2}\right) - \sqrt{1 - x^2}$ is :

(a) strictly increasing $\forall\, x \in (0, 1)$ (b) strictly decreasing $\forall\, x \in (-1, 0)$

(c) strictly decreasing for $x \in (-1, 0)$ (d) strictly decreasing for $x \in (0, 1)$

11. Let m and n be positive integers and $x, y > 0$ and $x + y = k$, where k is constant. Let $f(x, y) = x^m y^n$, then :

(a) $f(x, y)$ is maximum when $x = \frac{mk}{m + n}$

(b) $f(x, y)$ is maximum where $x = y$

(c) maximum value of $f(x, y)$ is $\frac{m^n n^m k^{m+n}}{(m + n)^{m+n}}$

(d) maximum value of $f(x, y)$ is $\frac{k^{m+n} m^m n^n}{(m + n)^{m+n}}$

12. The straight line which is both tangent and normal to the curve $x = 3t^2, y = 2t^3$ is :

(a) $y + \sqrt{3}(x - 1) = 0$ (b) $y - \sqrt{3}(x - 1) = 0$

(c) $y + \sqrt{2}(x - 2) = 0$ (d) $y - \sqrt{2}(x - 2) = 0$

13. A curve is such that the ratio of the subnormal at any point to the sum of its co-ordinates is equal to the ratio of the ordinate of this point to its abscissa. If the curve passes through (1, 0), then possible equation of the curve(s) is :

(a) $y = x\ln x$ (b) $y = \frac{\ln x}{x}$ (c) $y = \frac{2(x - 1)}{x^2}$ (d) $y = \frac{1 - x^2}{2x}$

14. A parabola of the form $y = ax^2 + bx + c\,(a > 0)$ intersects the graph of $f(x) = \frac{1}{x^2 - 4}$. The number of possible distinct intersection(s) of these graph can be :

(a) 0 (b) 2 (c) 3 (d) 4

15. Gradient of the line passing through the point (2, 8) and touching the curve $y = x^3$, can be :

(a) 3 (b) 6 (c) 9 (d) 12

16. The equation $x + \cos x = a$ has exactly one positive root, then :

(a) $a \in (0,1)$ (b) $a \in (2,3)$ (c) $a \in (1,\infty)$ (d) $a \in (-\infty,1)$

17. Given that $f(x)$ is a non-constant linear function. Then the curves :

(a) $y = f(x)$ and $y = f^{-1}(x)$ are orthogonal

(b) $y = f(x)$ and $y = f^{-1}(-x)$ are orthogonal

(c) $y = f(-x)$ and $y = f^{-1}(x)$ are orthogonal

(d) $y = f(-x)$ and $y = f^{-1}(-x)$ are orthogonal

18. Let $f(x) = \int_0^x e^{t^3}(t^2-1)t^2(t+1)^{2011}(t-2)^{2012}$ at $(x > 0)$ then :

(a) The number of point of inflections is atleast 1

(b) The number of point of inflections is 0

(c) The number of point of local maxima is 1

(d) The number of point of local minima is 1

19. Let $f(x) = \sin x + ax + b$. Then $f(x) = 0$ has :

(a) only one real root which is positive if $a > 1, b < 0$

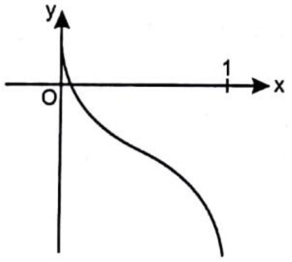
(b) only one real root which is negative if $a > 1, b > 0$

(c) only one real root which is negative if $a < -1, b < 0$

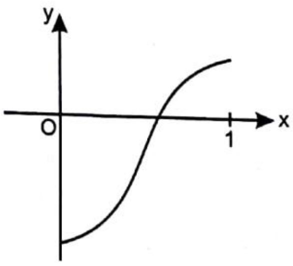
(d) only one real root which is positive if $a < -1, b < 0$

20. Which of the following graphs represent function whose derivatives have a maximum in the interval (0, 1) ?

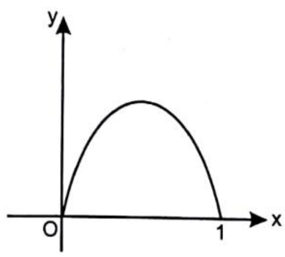
(a)



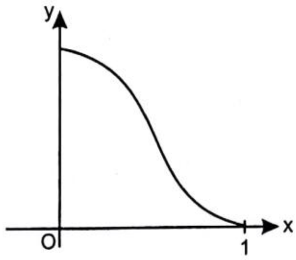
(b)



(c)



(d)



21. Consider $f(x) = \sin^5 x + \cos^5 x - 1,\ x \in \left[0, \frac{\pi}{2}\right]$, which of the following is/are correct ?

(a) f is strictly decreasing in $\left[0, \frac{\pi}{4}\right]$

(b) f is strictly increasing in $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

(c) There exist a number 'c' in $\left(0, \frac{\pi}{2}\right)$ such that $f'(c) = 0$

(d) The equation $f(x) = 0$ has only two roots in $\left[0, \frac{\pi}{2}\right]$

22. Let $f(x) = \begin{cases} x^{2\alpha+1} \ln x & ; \ x > 0 \\ 0 & ; \ x = 0 \end{cases}$

If $f(x)$ satisfies rolle's theorem in interval [0, 1], then α can be :

(a) $-\frac{1}{2}$ (b) $-\frac{1}{3}$ (c) $-\frac{1}{4}$ (d) -1

23. Which of the following is/are true for the function $f(x) = \int_0^x \frac{\cos t}{t} dt \ (x > 0)$?

(a) $f(x)$ is monotonically increasing in $\left((4n-1)\frac{\pi}{2}, (4n+1)\frac{\pi}{2}\right) \forall n \in N$

(b) $f(x)$ has a local minima at $x = (4n-1)\frac{\pi}{2} \ \forall n \in N$

(c) The points of inflection of the curve $y = f(x)$ lie on the curve $x \tan x + 1 = 0$

(d) Number of critical points of $y = f(x)$ in $(0, 10\pi)$ are 19

24. Let $F(x) = (f(x))^2 + (f'(x))^2$, $F(0) = 6$, where $f(x)$ is a thrice differentiable function such that $|f(x)| \le 1 \ \forall \ x \in [-1, 1]$, then choose the correct statement(s)

(a) there is atleast one point in each of the intervals (–1, 0) and (0, 1) where $|f'(x)| \le 2$

(b) there is atleast one point in each of the intervals (–1, 0) and (0, 1) where $F(x) \le 5$

(c) there is no point of local maxima of $F(x)$ in (–1, 1)

(d) for some $c \in (-1, 1)$, $F(c) \ge 6$, $F'(c) = 0$ and $F''(c) \le 0$

25. Let $f(x) = \begin{cases} x^3 + x^2 - 10x; & -1 \le x < 0 \\ \sin x; & 0 \le x < \frac{\pi}{2} \\ 1 + \cos x; & \frac{\pi}{2} \le x \le \pi \end{cases}$

then $f(x)$ has :

(a) local maximum at $x = \frac{\pi}{2}$

(b) local minimum at $x = \frac{\pi}{2}$

(c) absolute maximum at $x = 0$

(d) absolute maximum at $x = -1$

26. Minimum distance between the curves $y^2 = x - 1$ and $x^2 = y - 1$ is equal to :

(a) $\frac{\sqrt{2}}{4}$ (b) $\frac{3\sqrt{2}}{4}$ (c) $\frac{5\sqrt{2}}{4}$ (d) $\frac{7\sqrt{2}}{4}$

27. For the equation $\frac{e^{-x}}{1+x} = \lambda$ which of the following statement(s) is/are correct ?

(a) When $\lambda \in (0, \infty)$ equation has 2 real and distinct roots

(b) When $\lambda \in (-\infty, -e^2)$ equation has 2 real and distinct roots

(c) When $\lambda \in (0, \infty)$ equation has 1 real root

(d) When $\lambda \in (-e, 0)$ euqation has no real root

28. If $y = mx + 5$ is a tangent to the curve $x^3y^3 = ax^3 + by^3$ at $P(1, 2)$, then

(a) $a + b = \frac{18}{5}$ (b) $a > b$ (c) $a < b$ (d) $a + b = \frac{19}{5}$

29. If $(f(x) - 1)(x^2 + x + 1)^2 - (f(x) + 1)(x^4 + x^2 + 1) = 0$

$\forall x \in R - \{0\}$ and $f(x) \ne \pm 1$, then which of the following statement(s) is/are correct ?

(a) $|f(x)| \ge 2 \; \forall x \in R - \{0\}$

(b) $f(x)$ has a local maximum at $x = -1$

(c) $f(x)$ has a local minimum at $x = 1$

(d) $\int_{-\pi}^{\pi} (\cos x) f(x)\, dx = 0$

Answers

1.	(c, d)	**2.**	(a, c, d)	**3.**	(a, c)	**4.**	(a, c)	**5.**	(a, b, c, d)	**6.**	(b, c, d)
7.	(c, d)	**8.**	(b, d)	**9.**	(a, b, c, d)	**10.**	(a, c)	**11.**	(a, d)	**12.**	(c, d)
13.	(a, d)	**14.**	(b, c, d)	**15.**	(a, d)	**16.**	(b, c)	**17.**	(b, c)	**18.**	(a, d)
19.	(a, b, c)	**20.**	(a, b)	**21.**	(a, b, c, d)	**22.**	(b, c)	**23.**	(a, b, c)	**24.**	(a, b, d)
25.	(a, d)	**26.**	(b)	**27.**	(b, c, d)	**28.**	(a, d)	**29.**	(a, b, c, d)		

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

Let $y = f(x)$ such that $xy = x + y + 1, x \in R - \{1\}$ and $g(x) = xf(x)$

1. The minimum value of $g(x)$ is :

(a) $3-\sqrt{2}$ (b) $3+\sqrt{2}$ (c) $3-2\sqrt{2}$ (d) $3+2\sqrt{2}$

2. There exists two values of x, x_1 and x_2 where $g'(x) = \frac{1}{2}$, then $|x_1| + |x_2| =$

(a) 1 (b) 2 (c) 4 (d) 5

Paragraph for Question Nos. 3 to 5

Let $f(x) = \begin{cases} 1-x & ; \ 0 \le x \le 1 \\ 0 & ; \ 1 < x \le 2 \\ (2-x)^2 & ; \ 2 < x \le 3 \end{cases}$ and $g(x) = \int_0^x f(t)\,dt$.

Let the tangent to the curve $y = g(x)$ at point P whose abscissa is $\frac{5}{2}$ cuts x-axis in point Q.

Let the perpendicular from point Q on x-axis meets the curve $y = g(x)$ in point R.

3. $g(1) =$

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

4. Equation of tangent to the curve $y = g(x)$ at P is :

(a) $3y = 12x + 1$ (b) $3y = 12x - 1$ (c) $12y = 3x - 1$ (d) $12y = 3x + 1$

5. If 'θ' be the angle between tangents to the curve $y = g(x)$ at point P and R; then $\tan\theta$ equals to :

(a) $\frac{5}{6}$ (b) $\frac{5}{14}$ (c) $\frac{5}{7}$ (d) $\frac{5}{12}$

Paragraph for Question Nos. 6 to 8

Let $f(x) < 0 \ \forall \ x \in (-\infty, 0)$ and $f(x) > 0 \ \forall \ x \in (0, \infty)$ also $f(0) = 0$. Again $f'(x) < 0 \ \forall \ x \in (-\infty, -1)$ and $f'(x) > 0 \ \forall \ x \in (-1, \infty)$ also $f'(-1) = 0$ given $\lim_{x \to -\infty} f(x) = 0$ and $\lim_{x \to \infty} f(x) = \infty$ and function is twice differentiable.

6. If $f''(x) > 0 \ \forall \ x \in (-1, \infty)$ and $f'(0) = 1$ then number of solutions of equation $f(x) = x$ is :

(a) 2 (b) 3 (c) 4 (d) None of these

7. If $f''(x) < 0 \ \forall \ x \in (0, \infty)$ and $f'(0) = 1$ then number of solutions of equation $f(x) = x^2$ is :

(a) 1 (b) 2 (c) 3 (d) 4

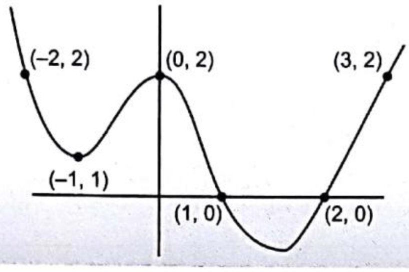
8. The minimum number of points where $f''(x)$ is zero is :

(a) 1 (b) 2 (c) 3 (d) 4

Paragraph for Question Nos. 9 to 11

In the given figure graph of :

$$y = p(x) = x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots\ldots + a_n \text{ is given.}$$



9. The product of all imaginary roots of $p(x) = 0$ is :

(a) −2 (b) −1 (c) −1/2 (d) none of these

10. If $p(x) + k = 0$ has 4 distinct real roots $\alpha, \beta, \gamma, \delta$ then $[\alpha] + [\beta] + [\gamma] + [\delta]$, (where $[\cdot]$ denotes greatest integer function) is equal to :

(a) −1 (b) −2 (c) 0 (d) 1

11. The minimum number of real roots of equation $(p'(x))^2 + p(x)\, p''(x) = 0$ are :

(a) 3 (b) 4 (c) 5 (d) 6

Paragraph for Question Nos. 12 to 14

The differentiable function $y = f(x)$ has a property that the chord joining any two points $A(x_1, f(x_1))$ and $B(x_2, f(x_2))$ always intersects y-axis at $(0, 2x_1 x_2)$. Given that $f(1) = -1$, then :

12. $\int_0^{1/2} f(x)\, dx$ is equal to :

(a) $\frac{1}{6}$ (b) $\frac{1}{8}$ (c) $\frac{1}{12}$ (d) $\frac{1}{24}$

13. The largest interval in which $f(x)$ is monotonically increasing, is :

(a) $\left(-\infty, \frac{1}{2}\right]$ (b) $\left[\frac{-1}{2}, \infty\right)$ (c) $\left(-\infty, \frac{1}{4}\right]$ (d) $\left[\frac{-1}{4}, \infty\right)$

14. In which of the following intervals, the Rolle's theorem is applicable to the function $F(x) = f(x) + x$?

(a) $[-1, 0]$ (b) $[0, 1]$ (c) $[-1, 1]$ (d) $[0, 2]$

Paragraph for Question Nos. 15 to 16

Let $f(x) = 1 + \int_0^1 (xe^y + ye^x)\, f(y)\, dy$ where x and y are independent variables.

15. If complete solution set of 'x' for which function $h(x) = f(x) + 3x$ is strictly increasing is $(-\infty, k)$ then $\left[\frac{4}{3}e^k\right]$ equals to : (where $[\cdot]$ denotes greatest integer function):

(a) 1 (b) 2 (c) 3 (d) 4

16. If acute angle of intersection of the curves $\frac{x}{2} + \frac{y}{3} + \frac{1}{3} = 0$ and $y = f(x)$ be θ then $\tan\theta$ equals to :

(a) $\frac{8}{25}$ (b) $\frac{16}{25}$ (c) $\frac{14}{25}$ (d) $\frac{4}{5}$

Answers

1.	(d)	**2.**	(c)	**3.**	(b)	**4.**	(c)	**5.**	(b)	**6.**	(d)	**7.**	(b)	**8.**	(a)	**9.**	(d)	**10.**	(a)
11.	(b)	**12.**	(d)	**13.**	(c)	**14.**	(b)	**15.**	(c)	**16.**	(a)								

Exercise-4 : Matching Type Problems

1. Column-I gives pair of curves and column-II gives the angle θ between the curves at their intersection point.

	Column-I		Column-II
(A)	$y = \sin x, y = \cos x$	(P)	$\frac{\pi}{4}$
(B)	$x^2 = 4y, y = \frac{8}{x^2+4}$	(Q)	$\frac{\pi}{2}$
(C)	$\frac{x^2}{18} + \frac{y^2}{8} = 1, x^2 - y^2 = 5$	(R)	$\tan^{-1} 3$
(D)	$xy = 1, x^2 - y^2 = 5$	(S)	$\tan^{-1} 5$
		(T)	$\tan^{-1}(2\sqrt{2})$

2.

	Column-I		Column-II
(A)	$(\sin^{-1} x)^{\cos^{-1} x} - (\cos^{-1} x)^{\sin^{-1} x} \forall x \in (\cos 1, \sin 1)$	(P)	Always positive
(B)	$(\cos x)^{\sin x} - (\sin x)^{\cos x} \forall x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$	(Q)	Always negative
(C)	$(\sin x)^{\sin x} - (\cos x)^{\sin x} \forall x \in \left(0, \frac{\pi}{2}\right)$	(R)	May be positive or negative for some values of x
(D)	$(\ln(\ln x))^{\ln(\ln x)} - (\ln x)^{\ln x} \forall x \in (e^e, \infty)$	(S)	May result in zero for some of values of x
		(T)	Indeterminate

3. Let $f(x) = \frac{x^3-4}{(x-1)^3} \forall x \neq 1$, $g(x) = \frac{x^4 - 2x^2}{4} \forall x \in R, h(x) \frac{x^3+4}{(x+1)^3} \forall x \neq -1$,

	Column-I		Column-II
(A)	The number of possible distinct real roots of equation $f(x) = c$ where $c \geq 4$ can be	(P)	0
(B)	The number of possible distinct real roots of equation $g(x) = c$, where $c \geq 0$ can be	(Q)	1
(C)	The number of possible distinct real roots of equation $h(x) = c$, where $c \geq 1$ can be	(R)	2

(D)	The number of possible distinct real roots of equation $g(x) = c$ where $-1 < c < 0$ can be	**(S)**	3
		(T)	4

4.

	Column-I		Column-II
(A)	If α, β, γ are roots of $x^3 - 3x^2 + 2x + 4 = 0$ and $y = 1 + \frac{\alpha}{x-\alpha} + \frac{\beta x}{(x-\alpha)(x-\beta)} + \frac{\gamma x^2}{(x-\alpha)(x-\beta)(x-\gamma)}$ then value of y at $x = 2$ is :	**(P)**	2
(B)	If $x^3 + ax + 1 = 0$ and $x^4 + ax + 1 = 0$ have a common roots then the value of $\|a\|$ can be equal to	**(Q)**	3
(C)	The number of local maximas of the function $x^2 + 4\cos x + 5$ is more than	**(R)**	4
(D)	If $f(x) = 2\|x\|^3 + 3x^2 - 12\|x\| + 1$, where $x \in [-1, 2]$ then greatest value of $f(x)$ is more than	**(S)**	5
		(T)	0

5.

	Column-I		Column-II
(A)	Maximum value of $f(x) = \log_2\left(\frac{4}{\sqrt{x+2}+\sqrt{2-x}}\right)$	**(P)**	0
(B)	The value of $\left[4\sum_{n=1}^{\infty}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right] =$ ([·] represent greatest integer function)	**(Q)**	1
(C)	Let $f(x) = x\sin \pi x, x > 0$ then number of points in $(0, 2)$ where $f'(x)$ vanishes, is	**(R)**	2
(D)	$\lim_{x\to 0^+}\left[\frac{x}{e^x - 1}\right] =$ ([·] represent greatest integer function)	**(S)**	3

6. Consider the function $f(x) = \frac{lnx}{8} - ax + x^2$ and $a \geq 0$ is a real constant :

	Column-I		Column-II
(A)	$f(x)$ gives a local maxima at	**(P)**	$a = 1;\ x = \frac{1}{4}$
(B)	$f(x)$ gives a local minima at	**(Q)**	$a > 1;\ x = \frac{a - \sqrt{a^2 - 1}}{4}$
(C)	$f(x)$ gives a point of inflection for	**(R)**	$0 \leq a < 1$
(D)	$f(x)$ is strictly increasing for all $x \in R^+$	**(S)**	$a > 1;\ x = \frac{a + \sqrt{a^2 - 1}}{4}$

7. The function $f(x) = \sqrt{ax^3 + bx^2 + cx + d}$ has its non-zero local minimum and maximum values at $x = -2$ and $x = 2$ respectively. If 'a' is one of the root of $x^2 - x - 6 = 0$, then match the following :

	Column-I		Column-II
(A)	The value of 'a' is	**(P)**	0
(B)	The value of 'b' is	**(Q)**	24
(C)	The value of 'c' is	**(R)**	Greater than 32
(D)	The value of 'd' is	**(S)**	−2

8.

	Column-I		Column-II
(A)	The ratio of altitude to the radius of the cylinder of maximum volume that can be inscribed in a given sphere is	**(P)**	$\frac{1}{\sqrt{2}}$
(B)	The ratio of radius to the altitude of the cone of the greatest volume which can be inscribed in a given sphere is	**(Q)**	$\sqrt{2}$
(C)	The cone circumscribing the sphere of radius 'r' has the maximum volume if its semi vertical angle is θ, then $33 \sin \theta =$	**(R)**	$\frac{32}{3}$
(D)	The greatest value of x^3y^4 if $2x + 3y = 7$, $x \geq 0, y \geq 0$ is	**(S)**	11

Answers

1.	$A \to T$; $B \to R$; $C \to Q$; $D \to Q$
2.	$A \to R, S$; $B \to Q$; $C \to R, S$; $D \to Q$
3.	$A \to Q, R$; $B \to R, S$; $C \to Q, R, S$; $D \to P, R, T$
4.	$A \to P$; $B \to P$; $C \to T$; $D \to P, Q, R, T$
5.	$A \to Q$; $B \to S$; $C \to R$; $D \to P$
6.	$A \to Q$; $B \to S$; $C \to P$; $D \to R$
7.	$A \to S$; $B \to P$; $C \to Q$; $D \to R$
8.	$A \to Q$; $B \to P$; $C \to S$; $D \to R$

Exercise-5 : Subjective Type Problems

1. A conical vessel is to be prepared out of a circular sheet of metal of unit radius. In order that the vessel has maximum volume, the sectorial area that must be removed from the sheet is A_1 and the area of the given sheet is A_2. If $\frac{A_2}{A_1} = m + \sqrt{n}$, where $m, n \in N$, then $m + n$ is equal to.

2. On [1, e], the least and greatest vlaues of $f(x) = x^2 \ln x$ are m and M respectively, then $[\sqrt{M + m}]$ is : (where [ ] denotes greatest integer function)

3. If $f(x) = \frac{px}{e^x} - \frac{x^2}{2} + x$ is a decreasing function for every $x \le 0$. Find the least value of p^2.

4. Let $f(x) = \begin{cases} xe^{ax} & , \ x \le 0 \\ x + ax^2 - x^3 & , \ x > 0 \end{cases}$. Where a is a positive constant. The interval in which $f'(x)$ is increasing is $\left[\frac{k}{a}, \frac{a}{l}\right]$. Then $k + l$ is equal to

5. Find sum of all possible values of the real parameter 'b' if the difference between the largest and smallest values of the function $f(x) = x^2 - 2bx + 1$ in the interval [0, 1] is 4.

6. Let 'θ' be the angle in radians between the curves $\frac{x^2}{36} + \frac{y^2}{4} = 1$ and $x^2 + y^2 = 12$. If $\theta = \tan^{-1}\left(\frac{a}{\sqrt{3}}\right)$; Find the value of a.

7. Let set of all possible values of λ such that $f(x) = e^{2x} - (\lambda + 1)e^x + 2x$ is monotonically increasing for $\forall x \in R$ is $(-\infty, k]$. Find the value of k.

8. Let a, b, c and d be non-negative real number such that $a^5 + b^5 \le 1$ and $c^5 + d^5 \le 1$. Find the maximum value of $a^2c^3 + b^2d^3$.

9. There is a point (p, q) on the graph of $f(x) = x^2$ and a point (r, s) on the graph of $g(x) = -8/x$, where $p > 0$ and $r > 0$. If the line through (p, q) and (r, s) is also tangent to both the curves at these points respectively, then find the value of $(p + r)$.

10. $f(x) = \max|2\sin y - x|$ where $y \in R$ then determine the minimum value of $f(x)$.

11. Let $f(x) = \int_0^x ((a-1)(t^2 + t + 1)^2 - (a+1)(t^4 + t^2 + 1))\,dt$. Then the total number of integral values of 'a' for which $f'(x) = 0$ has no real roots is

12. The number of real roots of the equation $x^{2013} + e^{2014x} = 0$ is

13. Let the maximum value of expression $y = \frac{x^4 - x^2}{x^6 + 2x^3 - 1}$ for $x > 1$ is $\frac{p}{q}$ where p and q are relatively prime positive integers. Find the value of $(p + q)$.

14. The least positive value of the parameter 'a' for which there exists atleast one line that is tangent to the graph of the curve $y = x^3 - ax$, at one point and normal to the graph at another point is $\frac{p}{q}$; where p and q are relatively prime positive integers. Find product pq.

15. Let $f(x) = x^2 + 2x - t^2$ and $f(x) = 0$ has two roots $\alpha(t)$ and $\beta(t)(\alpha < \beta)$ where t is a real parameter. Let $I(t) = \int_{\alpha}^{\beta} f(x)\,dx$. If the maximum value of $I(t)$ be λ and $|\lambda| = \frac{p}{q}$ where p and q are relatively prime positive integers. Find the product (pq).

16. A tank contains 100 litres of fresh water. A solution containing 1 gm/litre of salt runs into the tank at the rate of 1 lit/min. The homogenised mixture is pumped out of the tank at the rate of 3 lit/min. If T be the time when the amount of salt in the tank is maximum.
Find $[T]$ (where $[\cdot]$ denotes greatest integer function)

17. If $f(x)$ is continuous and differentiable in $[-3, 9]$ and $f'(x) \in [-2, 8]\ \forall\ x \in (-3, 9)$. Let N be the number of divisors of the greatest possible value of $f(9) - f(-3)$, then find the sum of digits of N.

18. It is given that $f(x)$ is defined on R satisfying $f(1) = 1$ and for $\forall\ x \in R$,
$f(x+5) \geq f(x) + 5$ and $f(x+1) \leq f(x) + 1$. If $g(x) = f(x) + 1 - x$, then $g(2002) =$

19. The number of normals to the curve $3y^3 = 4x$ which passes through the point $(0, 1)$ is

20. Find the number of real root(s) of the equation $ae^x = 1 + x + \frac{x^2}{2}$; where a is positive constant.

21. Let $f(x) = ax + \cos 2x + \sin x + \cos x$ is defined for $\forall\ x \in R$ and $a \in R$ and is strictly increasing function. If the range of a is $\left[\frac{m}{n}, \infty\right)$, then find the minimum value of $(m - n)$.

22. If p_1 and p_2 are the lengths of the perpendiculars from origin on the tangent and normal drawn to the curve $x^{2/3} + y^{2/3} = 6^{2/3}$ respectively. Find the value of $\sqrt{4p_1^2 + p_2^2}$.

Answers

1.	9	**2.**	2	**3.**	1	**4.**	1	**5.**	1	**6.**	2	**7.**	3
8.	1	**9.**	5	**10.**	2	**11.**	3	**12.**	1	**13.**	7	**14.**	12
15.	12	**16.**	27	**17.**	3	**18.**	1	**19.**	1	**20.**	1	**21.**	9
22.	6												

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Chapter 5 – Indefinite and Definite Integration

5 INDEFINITE AND DEFINITE INTEGRATION

Exercise-1 : Single Choice Problems

1. $\int a^x \left(\ln x + \ln a \cdot \ln \left(\frac{x}{e} \right)^x \right) dx =$

(a) $a^x \ln \left(\frac{e}{x} \right)^{2x} + C$

(b) $a^x \ln \left(\frac{x}{e} \right)^x + C$

(c) $a^x + \ln \left(\frac{x}{e} \right)^x + C$

(d) None of these

2. The value of :

$$\lim_{n \to \infty} \left(\frac{1}{\sqrt{n}\sqrt{n+1}} + \frac{1}{\sqrt{n}\sqrt{n+2}} + \frac{1}{\sqrt{n}\sqrt{n+3}} + \ldots\ldots + \frac{1}{\sqrt{n}\sqrt{2n}} \right) \text{ is :}$$

(a) $\sqrt{2} - 1$ (b) $2(\sqrt{2} - 1)$ (c) $\sqrt{2} + 1$ (d) $2(\sqrt{2} + 1)$

3. If $\int \frac{\sin x}{\sin (x - \alpha)} dx = Ax + B \log \sin (x - \alpha) + C$, then value of (A, B) is :

(a) $(\sin \alpha, \cos \alpha)$ (b) $(\cos \alpha, \sin \alpha)$ (c) $(-\sin \alpha, \cos \alpha)$ (d) $(-\cos \alpha, \sin \alpha)$

4. The value of the integral $\int_0^2 \frac{\log(x^2 + 2)}{(x + 2)^2} dx$ is :

(a) $\frac{\sqrt{2}}{3} \tan^{-1} \sqrt{2} + \frac{5}{12} \log 2 - \frac{1}{4} \log 3$

(b) $\frac{\sqrt{2}}{3} \tan^{-1} \sqrt{2} - \frac{5}{12} \log 2 - \frac{1}{12} \log 3$

(c) $\frac{\sqrt{2}}{3} \tan^{-1} \sqrt{2} + \frac{5}{12} \log 2 + \frac{1}{12} \log 3$

(d) $\frac{\sqrt{2}}{3} \tan^{-1} \sqrt{2} - \frac{5}{12} \log 2 + \frac{1}{12} \log 3$

5. If $I_1 = \int_0^1 \frac{1 + x^8}{1 + x^4} dx$ and $I_2 = \int_0^1 \frac{1 + x^9}{1 + x^3} dx$, then :

(a) $I_1 > 1, I_2 < 1$ (b) $I_1 < 1, I_2 > 1$ (c) $1 < I_1 < I_2$ (d) $I_2 < I_1 < 1$

6. Let $f:(0,1)\to(0,1)$ be a differentiable function such that $f'(x)\neq 0$ for all $x\in(0,1)$ and $f\left(\frac{1}{2}\right)=\frac{\sqrt{3}}{2}$. Suppose for all x, $\lim_{t\to x}\left(\frac{\int_0^t \sqrt{1-(f(s))^2}\,ds-\int_0^x \sqrt{1-(f(s))^2}\,ds}{f(t)-f(x)}\right)=f(x)$. Then the value of $f\left(\frac{1}{4}\right)$ belongs to :

(a) $\left\{\frac{\sqrt{7}}{4},\frac{\sqrt{15}}{4}\right\}$ (b) $\left\{\frac{\sqrt{7}}{3},\frac{\sqrt{15}}{3}\right\}$ (c) $\left\{\frac{\sqrt{7}}{2},\frac{\sqrt{15}}{2}\right\}$ (d) $\{\sqrt{7},\sqrt{15}\}$

7. If $f(\theta)=\frac{4}{3}(1-\cos^6\theta-\sin^6\theta)$, then

$$\lim_{n\to\infty}\frac{1}{n}\left[\sqrt{f\left(\frac{1}{n}\right)}+\sqrt{f\left(\frac{2}{n}\right)}+\sqrt{f\left(\frac{3}{n}\right)}+\ldots\ldots+\sqrt{f\left(\frac{n}{n}\right)}\right]=$$

(a) $\frac{1-\cos 1}{2}$ (b) $1-\cos 2$ (c) $\frac{\sin 2}{2}$ (d) $\frac{1-\cos 2}{2}$

8. The value of $\int_0^1 \frac{(x^6-x^3)}{(2x^3+1)^3}\,dx$ is equal to :

(a) $-\frac{1}{6}$ (b) $-\frac{1}{12}$ (c) $-\frac{1}{18}$ (d) $-\frac{1}{36}$

9. $2\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1}x}{x}\,dx-\int_0^1 \frac{\tan^{-1}x}{x}\,dx=$

(a) $\frac{\pi}{8}\ln 2$ (b) $\frac{\pi}{4}\ln 2$ (c) $\frac{\pi}{2\sqrt{2}}\ln 2$ (d) $\frac{\pi}{2}\ln 2$

10. Let $f(x)$ be a differentiable function such that $f(x)=x^2+\int_0^x e^{-t}f(x-t)\,dt$, then $\int_0^1 f(x)\,dx=$

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{7}{12}$ (d) $\frac{5}{12}$

11. If $f'(x)=f(x)+\int_0^1 f(x)\,dx$ and given $f(0)=1$, then $\int f(x)\,dx$ is equal to :

(a) $\frac{2}{3-e}e^x+\left(\frac{3-e}{1-e}\right)x+C$ (b) $\frac{2}{3-e}e^x+\left(\frac{1-e}{3-e}\right)x+C$

(c) $\frac{3}{2-e}e^x+\left(\frac{1+e}{3+e}\right)x+C$ (d) $\frac{2}{2-e}e^x+\left(\frac{1-e}{3+e}\right)x+C$

(where C is an arbitrary constant.)

12. For any $x \in R$, and f be a continuous function. Let $I_1 = \int_{\sin^2 x}^{1+\cos^2 x} tf(t(2-t))\,dt, I_2 = \int_{\sin^2 x}^{1+\cos^2 x} f(t(2-t))\,dt$, then $I_1 =$

(a) I_2 (b) $\frac{1}{2}I_2$ (c) $2I_2$ (d) $3I_2$

13. If the integral $\int \frac{5\tan x\,dx}{\tan x - 2} = x + a\ln|\sin x - 2\cos x| + C$, then '$a$' is equal to :

(a) 1 (b) 2 (c) −1 (d) −2

14. $\int \frac{(2+\sqrt{x})\,dx}{(x+1+\sqrt{x})^2}$ is equal to :

(a) $\frac{x}{x+\sqrt{x}+1}+C$ (b) $\frac{2x}{x+\sqrt{x}+1}+C$

(c) $\frac{-2x}{x+\sqrt{x}+1}+C$ (d) $\frac{-x}{x+\sqrt{x}+1}+C$

(where C is an arbitrary constant.)

15. Evaluate $\int \frac{\left(\sqrt[3]{x+\sqrt{2-x^2}}\right)\left(\sqrt[6]{1-x\sqrt{2-x^2}}\right)dx}{\sqrt[3]{1-x^2}}$; $x \in (0,1)$:

(a) $2^{\frac{1}{6}}x+C$ (b) $2^{\frac{1}{12}}x+C$

(c) $2^{\frac{1}{3}}x+C$ (d) None of these

16. $\int \frac{dx}{\sqrt{1-\tan^2 x}} = \frac{1}{\lambda}\sin^{-1}(\lambda \sin x)+C$, then $\lambda =$

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) 2 (d) $\sqrt{5}$

17. $\int \frac{dx}{\sqrt[3]{x^{5/2}(x+1)^{7/2}}}$ is equal to :

(a) $-\left(\frac{x+1}{x}\right)^{1/6}+C$ (b) $6\left(\frac{x+1}{x}\right)^{-1/6}+C$

(c) $\left(\frac{x}{x+1}\right)^{5/6}+C$ (d) $-\left(\frac{x}{x+1}\right)^{5/6}+C$

18. If $I_n = \int (\sin x)^n\,dx; n \in N$, then $5I_4 - 6I_6$ is equal to :

(a) $\sin x \cdot (\cos x)^5 + C$ (b) $\sin 2x \cos 2x + C$

(c) $\frac{\sin 2x}{8}[1+\cos^2 2x - 2\cos 2x]+C$ (d) $\frac{\sin 2x}{8}[1+\cos^2 2x + 2\cos 2x]+C$

19. $\int \frac{x^2}{(a+bx)^2} dx$ equals to :

(a) $\frac{1}{b^3}\left(a+bx-a\ln|a+bx|-\frac{a^2}{a+bx}\right)+C$

(b) $\frac{1}{b^3}\left(a+bx-2a\ln|a+bx|-\frac{a^2}{a+bx}\right)+C$

(c) $\frac{1}{b^3}\left(a+bx+2a\ln|a+bx|-\frac{a^2}{a+bx}\right)+C$

(d) $\frac{1}{b^3}\left(a+bx-2a\ln|a+ax|-\frac{a^2}{a+bx}\right)+C$

20. $\int \frac{8x^{43}+13x^{38}}{(x^{13}+x^5+1)^4} dx =$

(a) $\frac{x^{39}}{3(x^{13}+x^5+1)^3}+C$

(b) $\frac{x^{39}}{(x^{13}+x^5+1)^3}+C$

(c) $\frac{x^{39}}{5(x^{13}+x^5+1)^5}+C$

(d) None of these

21. $\int\left(\frac{\cos 6x+6\cos 4x+15\cos 2x+10}{10\cos^2 x+5\cos x\cos 3x+\cos x\cos 5x}\right)dx = f(x)+C$, then $f(10)$ is equal to :

(a) 20 (b) 10 (c) $2\sin 10$ (d) $2\cos 10$

22. $\int (1+x-x^{-1})e^{x+x^{-1}} dx =$

(a) $(x+1)e^{x+x^{-1}}+C$

(b) $(x-1)e^{x+x^{-1}}+C$

(c) $-xe^{x+x^{-1}}+C$

(d) $xe^{x+x^{-1}}+C$

23. If $\int e^x\left(\frac{2\tan x}{1+\tan x}+\text{cosec}^2\left(x+\frac{\pi}{4}\right)\right)dx = e^x\cdot g(x)+K$, then $g\left(\frac{5\pi}{4}\right) =$

(a) 0 (b) 1 (c) -1 (d) 2

24. $\int e^{x\sin x+\cos x}\left(\frac{x^4\cos^3 x - x\sin x+\cos x}{x^2\cos^2 x}\right)dx =$

(a) $e^{x\sin x+\cos x}\left(x-\frac{1}{\cos x}\right)+C$

(b) $e^{x\sin x+\cos x}\left(x-\frac{1}{x\cos x}\right)+C$

(c) $e^{x\sin x+\cos x}\left(1-\frac{1}{x\cos x}\right)+C$

(d) $e^{x\sin x+\cos x}\left(1-\frac{x}{\cos x}\right)+C$

25. The value of the definite integral $\int_0^1 \frac{1+x+\sqrt{x+x^2}dx}{\sqrt{x}+\sqrt{1+x}} dx$ is :

(a) $\frac{1}{3}(2^{1/2}-1)$

(b) $\frac{2}{3}(2^{1/2}-1)$

(c) $\frac{2}{3}(2^{3/2}-1)$

(d) $\frac{1}{3}(2^{3/2}-1)$

26. $\int x^{x^2+1}(2\ln x+1)\,dx$

(a) $x^{2x}+C$ (b) $x^2\ln x+C$ (c) $x^{(x^x)}+C$ (d) $(x^x)^x+C$

27. If $\int \frac{\operatorname{cosec}^2 x-2010}{\cos^{2010}x}dx=-\frac{f(x)}{(g(x))^{2010}}+C$; where $f\left(\frac{\pi}{4}\right)=1$; then the number of solutions of the equation $\frac{f(x)}{g(x)}=\{x\}$ in $[0,2\pi]$ is/are : (where $\{\cdot\}$ represents fractional part function)

(a) 0 (b) 1 (c) 2 (d) 3

28. $\int x^x\left((\ln x)^2+\ln x+\frac{1}{x}\right)dx$ is equal to :

(a) $x^x\left((\ln x)^2-\frac{1}{x}\right)+C$ (b) $x^x(\ln x-x)+C$

(c) $x^x\frac{(\ln x)^2}{2}+C$ (d) $x^x\ln x+C$

29. If $I=\int\frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}}$ is equal to :

(a) $\frac{\sqrt{2x^4-2x^2+1}}{x^2}+C$ (b) $\frac{\sqrt{2x^4-2x^2+1}}{x}+C$

(c) $\frac{\sqrt{2x^4-2x^2+1}}{x}+C$ (d) $\frac{\sqrt{2x^4-2x^2+1}}{2x^2}+C$

30. $I=\int\left(\frac{\ln x-1}{(\ln x)^2+1}\right)^2dx$ is equal to :

(a) $\frac{x}{x^2+1}+C$ (b) $\frac{\ln x}{(\ln x)^2+1}+C$ (c) $\frac{x}{1+(\ln x)^2}+C$ (d) $e^x\left(\frac{x}{x^2+1}\right)+C$

31. $I=\int\frac{dx}{\sqrt[4]{(x-1)^3(x+2)^5}}=k\sqrt[4]{\frac{x-1}{x+2}}+C$, then '$k$' is equal to :

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

32. $\int\frac{1-x^7}{x(1+x^7)}dx=P\log|x|+Q\log|x^7+1|+C$, then :

(a) $2P-7Q=0$ (b) $2P+7Q=0$ (c) $7P+2Q=0$ (d) $7P-2Q=1$

33. $I=\int\frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}dx$ is equal to :

(a) $\sin 2x+C$ (b) $\frac{\sin 2x}{2}+C$ (c) $\frac{-\sin 2x}{2}+C$ (d) $-2\sin 2x+C$

34. $I = \int \frac{(\sin 2x)^{1/3}\, d(\tan^{1/3} x)}{\sin^{2/3} x + \cos^{2/3} x} =$

(a) $\frac{1}{2^{2/3}} \ln(1 + \tan^{1/3} x) + C$ (b) $\ln(1 + \tan^{2/3} x) + C$

(c) $2^{1/3} \ln(1 + \tan^{2/3} x) + C$ (d) $\frac{1}{2^{2/3}} \ln(1 + \tan^{2/3} x) + C$

35. $\int \sqrt{\frac{(2012)^{2x}}{1-(2012)^{2x}}}\,(2012)^{\sin^{-1}(2012)^x}\, dx =$

(a) $(\log_{2012} e)^2 (2012)^{\sin^{-1}(2012)^x} + C$ (b) $(\log_{2012} e)^2 (2012)^{x+\sin^{-1}(2012)^x} + C$

(c) $(\log_{2012} e)^2 (2012)^{\sin^{-1}(2012)^x} + C$ (d) $\frac{(2012)^{\sin^{-1}(2012)^x}}{(\log_{2012} e)^2} + C$

(where C denotes arbitrary constant.)

36. $\int \frac{(x+2)\,dx}{(x^2+3x+3)\sqrt{x+1}}$ is equal to :

(a) $\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3(x+1)}}\right) + C$ (b) $\frac{2}{\sqrt{3}} \tan^{-1}\left(\sqrt{\frac{x}{3(x+1)}}\right) + C$

(c) $\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{\sqrt{x}}{3(x+1)}\right) + C$ (d) $\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}(x+1)}\right) + C$

(where C is arbitrary constant.)

37. $\int \left(\frac{f(x)g'(x) - f'(x)g(x)}{f(x)g(x)}\right)(\log(g(x)) - \log(f(x)))\,dx$ is equal to :

(a) $\log\left(\frac{g(x)}{f(x)}\right) + C$ (b) $\frac{1}{2}\left(\frac{g(x)}{f(x)}\right)^2 + C$

(c) $\frac{1}{2}\left(\log\left(\frac{g(x)}{f(x)}\right)\right)^2 + C$ (d) $\log\left(\left(\frac{g(x)}{f(x)}\right)^2\right) + C$

38. $\int \left(\int e^x \left(\ln x + \frac{2}{x} - \frac{1}{x^2}\right) dx\right) dx =$

(a) $e^x \ln x + C_1 x + C_2$ (b) $e^x \ln x + \frac{1}{x} + C_1 x + C_2$

(c) $\frac{\ln x}{x} + C_1 x + C_2$ (d) None of these

39. Maximum value of the function $f(x) = \pi^2 \int_0^1 t \sin(x + \pi t)dt$ over all real number x :

(a) $\sqrt{\pi^2 + 1}$ (b) $\sqrt{\pi^2 + 2}$ (c) $\sqrt{\pi^2 + 3}$ (d) $\sqrt{\pi^2 + 4}$

40. Let 'f' is a function, continuous on [0, 1] such that $f(x) \le \sqrt{5}\ \forall x \in [0, 1]$ and $f(x) \le \frac{2}{x} \forall x \in \left[\frac{1}{2}, 1\right]$ then the smallest 'a' for which $\int_0^1 f(x)\,dx \le a$ holds for all 'f' is :

(a) $\sqrt{5}$ (b) $\frac{\sqrt{5}}{2} + 2\ln 2$ (c) $2 + \ln\left(\frac{\sqrt{5}}{2}\right)$ (d) $2 + 2\ln\left(\frac{\sqrt{5}}{2}\right)$

41. Let $I_n = \int_1^{e^2} (\ln x)^n\, d(x^2)$, then the value of $2I_n + nI_{n-1}$ equals to :

(a) 0 (b) $2e^2$ (c) e^2 (d) 1

42. Let a function $f: R \to R$ be defined as $f(x) = x + \sin x$. The value of $\int_0^{2\pi} f^{-1}(x)\,dx$ will be :

(a) $2\pi^2$ (b) $2\pi^2 - 2$ (c) $2\pi^2 + 2$ (d) π^2

43. The value of the definite integral $\int_{-1}^{1} e^{-x^4}\left(2 + \ln\left(x + \sqrt{x^2 + 1}\right) + 5x^3 - 8x^4\right)dx$ is equal to :

(a) $4e$ (b) $\frac{4}{e}$ (c) $2e$ (d) $\frac{2}{e}$

44. $\int_{-10}^{0} \frac{\left|\frac{2[x]}{3x - [x]}\right|}{\frac{2[x]}{3x - [x]}} dx$ is equal to (where [*] denotes greatest integer function.)

(a) $\frac{28}{3}$ (b) $\frac{1}{3}$ (c) 0 (d) None of these

45. If $f(x) = \frac{x}{1 + (\ln x)(\ln x)\ldots\infty}\ \forall x \in [1, \infty)$ then $\int_1^{2e} f(x)\,dx$ equals is :

(a) $\frac{e^2 - 1}{2}$ (b) $\frac{e^2 + 1}{2}$ (c) $\frac{e^2 - 2e}{2}$ (d) None of these

46. $\int_0^4 \frac{(y^2 - 4y + 5)\sin(y - 2)}{(2y^2 - 8y + 11)} dy$ is equal to :

(a) 0 (b) 2 (c) -2 (d) None of these

47. Let $\frac{d}{dx}F(x)=\left(\frac{e^{\sin x}}{x}\right)$, $x>0$. If $\int_1^4 \frac{3}{x}e^{\sin x^3}dx=F(k)-F(1)$, then one of the possible values of k, is:

(a) 15 (b) 16 (c) 63 (d) 64

48. Value of $\lim_{h\to 0}\frac{\int_0^{\pi+he^{-1/h}} x^2e^{-x^2}dx-\int_0^{\pi} x^2e^{-x^2}dx}{he^{-1/h}}$ is equal to :

(a) $\pi(1-\pi^2)e^{-\pi^2}$ (b) $2\pi(1-\pi^2)e^{-\pi^2}$ (c) $\pi(1-\pi)e^{-\pi}$ (d) $\pi^2e^{-\pi^2}$

49. Let $f:R^+\to R$ be a differentiable function with $f(1)=3$ and satisfying :

$\int_1^{xy} f(t)\,dt=y\int_1^x f(t)\,dt+x\int_1^y f(t)\,dt\ \forall\, x,y\in R^+$, then $f(e)=$

(a) 3 (b) 4 (c) 1 (d) None of these

50. If [·] denotes the greatest integer function, then the integral $\int_0^{\pi/2}\frac{e^{\sin x-[\sin x]}d(\sin^2 x-[\sin^2 x])}{\sin x-[\sin x]}$ is λ, then $[\lambda-1]$ is equal to :

(a) 0 (b) 1 (c) 2 (d) 3

51. Calculate the reciprocal of the limit $\lim_{x\to\infty}\int_0^x xe^{t^2-x^2}dt$

(a) 0 (b) 1 (c) 2 (d) 3

52. Let $L=\lim_{n\to\infty}\left(\frac{(2\cdot1+n)}{1^2+n\cdot1+n^2}+\frac{(2\cdot2+n)}{2^2+n\cdot2+n^2}+\frac{(2\cdot3+n)}{3^2+n\cdot3+n^2}+\ldots\ldots+\frac{(2\cdot n+n)}{3n^2}\right)$ then value of e^L is :

(a) 2 (b) 3 (c) 4 (d) $\frac{3}{2}$

53. The value of the definite integral $\int_0^2\left(\sqrt{1+x^3}+\sqrt[3]{x^2+2x}\right)dx$ is :

(a) 4 (b) 5 (c) 6 (d) 7

54. The value of the definite integral $\int_0^\infty \frac{\ln x}{x^2+4}dx$ is :

(a) $\frac{\pi\ln 3}{2}$ (b) $\frac{\pi\ln 2}{3}$

(c) $\frac{\pi\ln 2}{4}$ (d) $\frac{\pi\ln 4}{3}$

55. The value of the definite integral $\int_0^{10}((x-5)+(x-5)^2+(x-5)^3)\,dx$ is :

(a) $\frac{125}{3}$ (b) $\frac{250}{3}$ (c) $\frac{125}{6}$ (d) $\frac{250}{4}$

56. The value of definite integral $\int_0^{\infty}\frac{dx}{(1+x^9)(1+x^2)}$ equals to :

(a) $\frac{\pi}{16}$ (b) $\frac{\pi}{8}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

57. The value of the definite integral $\int_0^{\pi/2}\left(\frac{1+\sin 3x}{1+2\sin x}\right)dx$ equals to :

(a) $\frac{\pi}{2}$ (b) 1 (c) $\frac{1}{2}$ (d) $\frac{\pi}{4}$

58. The value of $\lim_{x\to\infty}\frac{\int_0^x(\tan^{-1}x)^2\,dx}{\sqrt{x^2+1}}=$

(a) $\frac{\pi^2}{16}$ (b) $\frac{\pi^2}{4}$

(c) $\frac{\pi^2}{2}$ (d) None of these

59. If $\int_0^1\left(\sum_{r=1}^{2013}\frac{x}{x^2+r^2}\right)\left(\prod_{r=1}^{2013}(x^2+r^2)\right)dx=\frac{1}{2}\left[\left(\prod_{r=1}^{2013}(1+r^2)\right)-k^2\right]$

then $k=$

(a) 2013 (b) 2013! (c) 2013^2 (d) 2013^{2013}

60. $f(x)=2x-\tan^{-1}x-\ln(x+\sqrt{1+x^2})$

(a) strictly increases $\forall\, x\in R$

(b) strictly increases only in $(0,\infty)$

(c) strictly decreases $\forall\, x\in R$

(d) strictly decreases in $(0,\infty)$ and strictly increases in $(-\infty,0)$

61. The value of the definite integral $\int_0^{\pi/2}\frac{dx}{\tan x+\cot x+\operatorname{cosec} x+\sec x}$ is :

(a) $1-\frac{\pi}{4}$ (b) $\frac{\pi}{4}+1$ (c) $\pi+\frac{1}{4}$ (d) None of these

62. The value of the definite integral $\int_3^7 \frac{\cos x^2}{\cos x^2 + \cos(10-x)^2}\,dx$ is :

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) None of these

63. The value of the integral $\int_{e^{-1}}^{e^2} \left|\frac{\ln x}{x}\right| dx$ is :

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) 3 (d) 5

64. The value of $\lim_{x\to\frac{\pi}{4}} \frac{\int_2^{\mathrm{cosec}^2 x} tg(t)\,dt}{x^2 - \frac{\pi^2}{16}}$ is :

(a) $\frac{2}{\pi}g(2)$ (b) $-\frac{4}{\pi}g(2)$ (c) $-\frac{16}{\pi}g(2)$ (d) $-4g(2)$

65. The value of $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{n-k}{n^2} \cos\frac{4k}{n}$ equals :

(a) $\frac{1}{4}\sin 4 + \frac{1}{16}\cos 4 - \frac{1}{16}$

(b) $\frac{1}{4}\sin 4 - \frac{1}{16}\cos 4 + \frac{1}{16}$

(c) $\frac{1}{16}(1 - \sin 4)$

(d) $\frac{1}{16}(1 - \cos 4)$

66. For each positive integer n, define a function f_n on [0, 1] as follows :

$$f_n(x) = \begin{cases} 0 & \text{if} \quad x = 0 \\ \sin\frac{\pi}{2n} & \text{if} \quad 0 < x \le \frac{1}{n} \\ \sin\frac{2\pi}{2n} & \text{if} \quad \frac{1}{n} < x \le \frac{2}{n} \\ \sin\frac{3\pi}{2n} & \text{if} \quad \frac{2}{n} < x \le \frac{3}{n} \\ \sin\frac{n\pi}{2n} & \text{if} \quad \frac{n-1}{n} < x \le 1 \end{cases}$$

Then the value of $\lim_{n\to\infty} \int_0^1 f_n(x)\,dx$ is :

(a) π

(b) $\frac{\pi}{2}$

(c) $\frac{1}{\pi}$

(d) $\frac{2}{\pi}$

67. Let n be a positive integer, then

$$\int_0^{n+1} \min\{|x-1|,|x-2|,|x-3|,\ldots\ldots|x-n|\}\,dx \text{ equals}$$

(a) $\frac{(n+1)}{4}$ (b) $\frac{(n+2)}{4}$ (c) $\frac{(n+3)}{4}$ (d) $\frac{(n+4)}{4}$

68. For positive integers $k = 1, 2, 3\ldots\ldots, n$, let S_k denotes the area of ΔAOB_k (where 'O' is origin) such that $\angle AOB_k = \frac{k\pi}{2n}, OA = 1$ and $OB_k = k$. The value of the $\lim_{n\to\infty} \frac{1}{n^2}\sum_{k=1}^{n} S_k$ is :

(a) $\frac{2}{\pi^2}$ (b) $\frac{4}{\pi^2}$ (c) $\frac{8}{\pi^2}$ (d) $\frac{1}{2\pi^2}$

69. If $A = \int_0^1 \prod_{r=1}^{2014}(r-x)\,dx$ and $B = \int_0^1 \prod_{r=0}^{2013}(r+x)\,dx$, then :

(a) $A = 2B$ (b) $2A = B$ (c) $A + B = 0$ (d) $A = B$

70. If $f(x) = \left[\frac{x}{120} + \frac{x^3}{30}\right]$ defined in $[0, 3]$, then $\int_0^1 (f(x)+2)\,dx =$

(where $[\cdot]$ denotes greatest integer function)

(a) 0 (b) 1 (c) 2 (d) 4

71. If $f(x) = \int_0^{g(x)} \frac{dt}{\sqrt{1+t^3}}, g(x) = \int_0^{\cos x}(1+\sin t)^2\,dt$, then the value of $f'\left(\frac{\pi}{2}\right)$ is equal to :

(a) 1 (b) −1 (c) 0 (d) $\frac{1}{2}$

72. Let $f(x) = \frac{1}{x^2}\int_0^x (4t^2 - 2f'(t))\,dt$, find $9f'(4)$

(a) 16 (b) 4 (c) 8 (d) 32

73. Evaluate $\lim_{n\to\infty}\left(\frac{1^2}{n^3+1^3} + \frac{2^2}{n^3+2^3} + \frac{3^2}{n^3+3^3} + \ldots\ldots + \frac{4}{9n}\right)$.

(a) $\frac{1}{3}\ln 3$ (b) $\frac{\ln 9}{3}$ (c) $\frac{\ln 4}{3}$ (d) $\frac{\ln 6}{3}$

74. The value of $\int_0^{2\pi} \cos^{-1}\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right) dx$ is :

(a) π^2 (b) $\frac{\pi^2}{2}$ (c) $2\pi^2$ (d) π^3

75. Given a function 'g' continuous everywhere such that $\int_0^1 g(t)\,dt = 2$ and $g(1) = 5$.

If $f(x) = \frac{1}{2}\int_0^x (x-t)^2 g(t)\,dt$, then the value of $f'''(1) - f''(1)$ is :

(a) 0 (b) 1 (c) 2 (d) 3

76. If $\int_0^{\pi} \frac{x^3 \cos^4 x \sin^2 x}{\pi^2 - 3\pi x + 3x^2}\,dx = \lambda \int_0^{\pi/2} \sin^2 x\,dx$, then the value of λ is :

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{8}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

77. $\int_0^{\sqrt{3}} \left(\frac{1}{2}\frac{d}{dx}\left(\tan^{-1}\frac{2x}{1-x^2}\right)\right)dx$ equals to :

(a) $\frac{\pi}{3}$ (b) $-\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) None of these

78. Let $y = \{x\}^{[x]}$ then the value of $\int_0^3 y\,dx$ equals to :

(where $\{\cdot\}$ and $[\cdot]$ denote fractional part and greatest integer function respectively.)

(a) 1 (b) $\frac{11}{6}$ (c) 3 (d) $\frac{5}{6}$

79. $\int_0^1 \frac{\tan^{-1} x}{x}\,dx =$

(a) $\int_0^{\pi/4} \frac{\sin x}{x}\,dx$ (b) $\int_0^{\pi/2} \frac{x}{\sin x}\,dx$ (c) $\frac{1}{2}\int_0^{\pi/2} \frac{x}{\sin x}\,dx$ (d) $\frac{1}{2}\int_0^{\pi/4} \frac{x}{\sin x}\,dx$

80. The value of $\int_0^{4/\pi} \left(3x^2 \sin\frac{1}{x} - x\cos\frac{1}{x}\right)dx$ is :

(a) $\frac{8\sqrt{2}}{\pi^3}$ (b) $\frac{24\sqrt{2}}{\pi^3}$

(c) $\frac{32\sqrt{2}}{\pi^3}$ (d) None of these

81. The number of values of x satisfying the equation :

$$\int_{-1}^{x} \left(8t^2 + \frac{28t}{3} + 4\right)dt = \frac{\frac{3}{2}x + 1}{\log_{(x+1)} \sqrt{x+1}}, \text{ is :}$$

(a) 0 (b) 1 (c) 2 (d) 3

82. $\lim_{n\to\infty} \frac{1+2^4+3^4+\ldots\ldots+n^4}{n^5} - \lim_{n\to\infty} \frac{1+2^3+3^3+\ldots\ldots+n^3}{n^5}$ is :

(a) $\frac{1}{30}$ (b) zero (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

83. The value of $\lim_{x\to 0^+} \frac{\int_1^{\cos x} (\cos^{-1} t)\, dt}{2x - \sin 2x}$ is equal to :

(a) 0 (b) -1 (c) $\frac{2}{3}$ (d) $-\frac{1}{4}$

84. Consider a parabola $y = \frac{x^2}{4}$ and the point $F(0,1)$.

Let $A_1(x_1, y_1), A_2(x_2, y_2), A_3(x_3, y_3), \ldots\ldots, A_n(x_n, y_n)$ are 'n' points on the parabola such $x_k > 0$ and $\angle OFA_k = \frac{k\pi}{2n}(k = 1, 2, 3, \ldots\ldots, n)$. Then the value of $\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} FA_k$, is equal to :

(a) $\frac{2}{\pi}$ (b) $\frac{4}{\pi}$ (c) $\frac{8}{\pi}$ (d) None of these

85. The minimum value of $f(x) = \int_0^4 e^{|x-t|}\, dt$ where $x \in [0, 3]$ is :

(a) $2e^2 - 1$ (b) $e^4 - 1$ (c) $2(e^2 - 1)$ (d) $e^2 - 1$

86. If $\int_0^\infty \frac{\cos x}{x} dx = \frac{\pi}{2}$, then $\int_0^\infty \frac{\cos^3 x}{x} dx$ is equals to :

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) π (d) $\frac{3\pi}{2}$

87. $\int \sqrt{1+\sin x}\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right) dx =$:

(a) $\frac{1+\sin x}{2} + C$ (b) $(1+\sin x)^2 + C$ (c) $\frac{1}{\sqrt{1+\sin x}} + C$ (d) $\sin x + C$

88. If $I_n = \int_0^\pi \frac{\sin(2nx)}{\sin 2x} dx$, then the value of $I_{n+\frac{1}{2}}$ is equal to $(n \in I)$:

(a) $\frac{n\pi}{2}$ (b) π (c) $\frac{\pi}{2}$ (d) 0

89. The value of function $f(x) = 1 + x + \int_1^x (ln^2 t + 2\, ln\, t)\, dt$ where $f'(x)$ vanishes is :

(a) $\frac{1}{e}$ (b) 0 (c) $\frac{2}{e}$ (d) $1 + \frac{2}{e}$

90. Let f be a differentiable function on R and satisfies $f(x) = x^2 + \int_0^x e^{-t} f(x-t)dt$; then $\int_0^1 f(x)\, dx$ is equal to :

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{7}{12}$ (d) $\frac{5}{12}$

91. The value of the definite integral $\int_{-(\pi/2)}^{\pi/2} \frac{\cos^2 x}{1+5^x}$ equals to :

(a) $\frac{3\pi}{4}$ (b) π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

92. $\int \left(\frac{x^2 - x + 1}{x^2 + 1}\right) e^{\cot^{-1}(x)} dx = f(x) \cdot e^{\cot^{-1}(x)} + C$

where C is constant of integration. Then $f(x)$ is equal to :

(a) $-x$ (b) $\sqrt{1-x}$ (c) x (d) $\sqrt{1+x}$

93. $\lim_{n\to\infty} \frac{1}{n^3}(\sqrt{n^2+1} + 2\sqrt{n^2+2^2} + \ldots\ldots + n\sqrt{(n^2+n^2)}) = $:

(a) $\frac{3\sqrt{2}-1}{2}$ (b) $\frac{2\sqrt{2}-1}{3}$ (c) $\frac{3\sqrt{3}-1}{3}$ (d) $\frac{4\sqrt{2}-1}{2}$

94. $\int \frac{(x^3-1)}{(x^4+1)(x+1)} dx$, is :

(a) $\frac{1}{4}\ln(1+x^4) + \frac{1}{3}\ln(1+x^3) + c$ (b) $\frac{1}{4}\ln(1+x^4) - \frac{1}{3}\ln(1+x^3) + c$

(c) $\frac{1}{4}\ln(1+x^4) - \ln(1+x) + c$ (d) $\frac{1}{4}\ln(1+x^4) + \ln(1+x) + c$

95. The value of Limit $\lim_{x\to 0^+} \frac{\int_1^{\cos x} (\cos^{-1} t)\, dt}{2x - \sin 2x}$ is equal to :

(a) 0 (b) -1 (c) $\frac{2}{3}$ (d) $\frac{-1}{4}$

96. Let $f(x) = \lim_{n\to\infty} \frac{\cos x}{1 + (\tan^{-1} x)^n}$, then $\int_0^\infty f(x)\, dx =$

(a) $\tan(\sin 1)$ (b) $\sin(\tan 1)$ (c) 0 (d) $\sin\left(\frac{\tan 1}{2}\right)$

97. The value of $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{k}{n^2 + n + 2k}\right) =$

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 1

98. The value of $\lim_{y\to 1^+} \frac{\int_1^y |t-1|\,dt}{\tan(y-1)}$ is :

(a) 0 (b) 1 (c) 2 (d) does not exist

99. Given that $\int \frac{dx}{(1+x^2)^n} = \frac{x}{2(n-1)(1+x^2)^{n-1}} + \frac{(2n-3)}{2(n-1)}\int \frac{dx}{(1+x^2)^{n-1}}$. Find the value of $\int_0^1 \frac{dx}{(1+x^2)^4}$: (you may or may not use reduction formula given)

(a) $\frac{11}{48}+\frac{5\pi}{64}$ (b) $\frac{11}{48}+\frac{5\pi}{32}$ (c) $\frac{1}{24}+\frac{5\pi}{64}$ (d) $\frac{1}{96}+\frac{5\pi}{32}$

100. Find the value of $\int_0^{\pi/4} (\sin x)^4\,dx$:

(a) $\frac{3\pi}{16}$ (b) $\frac{3\pi}{32}-\frac{1}{4}$ (c) $\frac{3\pi}{32}-\frac{3}{4}$ (d) $\frac{3\pi}{16}-\frac{7}{8}$

101. $\int \frac{\cos 9x + \cos 6x}{2\cos 5x - 1}\,dx = A\sin 4x + B\sin + C$, then $A+B$ is equal to :

(Where C is constant of integration)

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) 2 (d) $\frac{5}{4}$

102. $\int \frac{dx}{x^{2014}+x} = \frac{1}{p}\ln\left(\frac{x^q}{1+x^r}\right) + C$ where $p, q, r \in N$ then the value of $(p+q+r)$ equals

(Where C is constant of integration)

(a) 6039 (b) 6048 (c) 6047 (d) 6021

103. If $\int_0^1 e^{-x^2}\,dx = a$, then $\int_0^1 x^2 e^{-x^2}\,dx$ is equal to

(a) $\frac{1}{2e}(ea-1)$ (b) $\frac{1}{2e}(ea+1)$ (c) $\frac{1}{e}(ea-1)$ (d) $\frac{1}{e}(ea+1)$

104. If $f(x)$ is a continuous function for all real values of x and satisfies $\int_n^{n+1} f(x)\,dx = \frac{n^2}{2}\ \forall\, n \in I$, then $\int_{-3}^{5} f(|x|)\,dx$ is equal to :

(a) $\frac{19}{2}$ (b) $\frac{35}{2}$ (c) $\frac{17}{2}$ (d) $\frac{37}{2}$

105. If $\int \frac{dx}{x^4(1+x^3)^2} = a\ln\left|\frac{1+x^3}{x^3}\right| + \frac{b}{x^3} + \frac{c}{1+x^3} + d$, then

(where d is arbitrary constant)

(a) $a = \frac{1}{3}, b = \frac{1}{3}, x = \frac{1}{3}$

(b) $a = \frac{2}{3}, b = -\frac{1}{3}, c = \frac{1}{3}$

(c) $a = \frac{2}{3}, b = -\frac{1}{3}, c = -\frac{1}{3}$

(d) $a = \frac{2}{3}, b = \frac{1}{3}, c = -\frac{1}{3}$

106. $\lim_{n\to\infty} \frac{1}{\sqrt{n}\sqrt{n+1}} + \frac{1}{\sqrt{n}\sqrt{n+2}} + \ldots\ldots + \frac{1}{\sqrt{n}\sqrt{4n}}$ is equal to :

(a) 2 (b) 4 (c) $2(\sqrt{2}-1)$ (d) $2\sqrt{2}-1$

107. Let $f(x) = \int_x^2 \frac{dy}{\sqrt{1+y^3}}$. The value of the integral $\int_0^2 xf(x)\,dx$ is equal to :

(a) 1 (b) $\frac{1}{3}$ (c) $\frac{4}{3}$ (d) $\frac{2}{3}$

108. The value of the definite integral $\int_0^{\pi/3} \ln(1+\sqrt{3}\tan x)\,dx$ equals

(a) $\frac{\pi}{3}\ln 2$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi^2}{6}\ln 2$ (d) $\frac{\pi}{2}\ln 2$

109. If $\int_0^{100} f(x)\,dx = a$, then $\sum_{r=1}^{100}\int_0^1 (f(r-1+x)\,dx) =$

(a) $100a$ (b) a (c) 0 (d) $10a$

110. The value of $\int_0^1 \lim_{n\to\infty} \sum_{k=0}^{n} \frac{x^{k+2}2^k}{k!}\,dx$ is :

(a) e^2-1 (b) 2 (c) $\frac{e^2-1}{2}$ (d) $\frac{e^2-1}{4}$

111. Evaluate : $\int x^5\sqrt{1+x^3}\,dx$.

(a) $\frac{1}{15}(1+x^3)^{5/2} - \frac{1}{9}(1+x)^3)^{3/2} + c$

(b) $\frac{2}{15}(1+x^3)^{5/2} - \frac{1}{9}(1+x^3)^{3/2} + c$

(c) $\frac{2}{15}(1+x^3)^{5/2} - \frac{2}{9}(1+x^3)^{3/2} + c$

(d) $\frac{1}{15}(1+x^3)^{5/2} - \frac{2}{9}(1+x^3)^{3/2} + c$

112. If $f(x) = \int_0^x \frac{\sin t}{t}\, dt$, which of the following is true ?

(a) $f(0) > f(1\cdot 1)$

(b) $f(0) < f(1\cdot 1) > f(2\cdot 1)$

(c) $f(0) < f(1\cdot 1) < f(2\cdot 1) > f(3\cdot 1)$

(d) $f(0) < f(1\cdot 1) < f(2\cdot 1) < f(3\cdot 1) > f(4\cdot 1)$

113. Evaluate : $\int \frac{x^3 + 3x^2 + x + 9}{(x^2+1)(x^2+3)} dx.$

(a) $\ln|x^2 + 3| + 3\tan^{-1} x + c$

(b) $\frac{1}{2}\ln|x^2 + 3| + \tan^{-1} x + c$

(c) $\frac{1}{2}\ln|x^2 + 3| + 3\tan^{-1} x + c$

(d) $\ln|x^2 + 3| - \tan^{-1} x + c$

114. $\int \frac{\sqrt{\sec^5 x}}{\sqrt{\sin^3 x}} dx$ equals to :

(a) $(\tan x)^{3/2} - \sqrt{\tan x} + C$

(b) $2\left(\frac{1}{3}(\tan x)^{3/2} - \frac{1}{\sqrt{\tan x}}\right) + C$

(c) $\frac{1}{3}(\tan x)^{3/2} - \sqrt{\tan x} + C$

(d) $\sqrt{\sin x} + \sqrt{\cos x} + C$

115. $\lim_{x\to 0} \int_0^x \frac{e^{\sin(tx)}}{x} dt$ equals to :

(a) 1 (b) 2 (c) e (d) Does not exist

116. If $A = \int_0^{\pi} \frac{\sin x}{x^2} dx$, then $\int_0^{\pi/2} \frac{\cos 2x}{x} dx$ is equal to :

(a) $1 - A$ (b) $\frac{3}{2} - A$ (c) $A - 1$ (d) $1 + A$

Answers

1. (b)	**2.** (b)	**3.** (b)	**4.** (d)	**5.** (d)	**6.** (a)	**7.** (d)	**8.** (d)	**9.** (b)	**10.** (d)
11. (b)	**12.** (a)	**13.** (b)	**14.** (b)	**15.** (a)	**16.** (a)	**17.** (b)	**18.** (c)	**19.** (b)	**20.** (a)
21. (a)	**22.** (d)	**23.** (b)	**24.** (b)	**25.** (c)	**26.** (d)	**27.** (a)	**28.** (d)	**29.** (d)	**30.** (c)
31. (d)	**32.** (b)	**33.** (c)	**34.** (d)	**35.** (c)	**36.** (a)	**37.** (c)	**38.** (a)	**39.** (d)	**40.** (d)
41. (b)	**42.** (a)	**43.** (b)	**44.** (a)	**45.** (a)	**46.** (a)	**47.** (d)	**48.** (d)	**49.** (d)	**50.** (c)
51. (c)	**52.** (b)	**53.** (c)	**54.** (c)	**55.** (b)	**56.** (c)	**57.** (b)	**58.** (b)	**59.** (b)	**60.** (a)
61. (a)	**62.** (a)	**63.** (b)	**64.** (c)	**65.** (d)	**66.** (d)	**67.** (a)	**68.** (d)	**69.** (d)	**70.** (b)
71. (d)	**72.** (b)	**73.** (a)	**74.** (d)	**75.** (b)	**76.** (a)	**77.** (b)	**78.** (c)	**79.** (c)	**80.** (c)
81. (b)	**82.** (d)	**83.** (d)	**84.** (b)	**85.** (c)	**86.** (a)	**87.** (d)	**88.** (d)	**89.** (d)	**90.** (d)
91. (d)	**92.** (c)	**93.** (b)	**94.** (c)	**95.** (d)	**96.** (b)	**97.** (c)	**98.** (a)	**99.** (a)	**100.** (b)
101. (d)	**102.** (a)	**103.** (a)	**104.** (b)	**105.** (c)	**106.** (a)	**107.** (d)	**108.** (a)	**109.** (b)	**110.** (d)
111. (c)	**112.** (d)	**113.** (c)	**114.** (b)	**115.** (a)	**116.** (c)				

Exercise-2 : One or More than One Answer is/are Correct

1. $\int \frac{dx}{(1+\sqrt{x})^8} = -\frac{1}{3(1+\sqrt{x})^{k_1}} + \frac{2}{7(1+\sqrt{x})^{k_2}} + C$, then :

(a) $k_1 = 5$ (b) $k_1 = 6$ (c) $k_2 = 7$ (d) $k_2 = 8$

2. If $\int_{-\alpha}^{\alpha}\left(e^x + \cos x \ln\left(x+\sqrt{1+x^2}\right)\right)dx > \frac{3}{2}$, then possible value of α can be :

(a) 1 (b) 2 (c) 3 (d) 4

3. For $a > 0$, if $I = \int \sqrt{\frac{x}{a^3 - x^3}}dx = A\sin^{-1}\left(\frac{x^{3/2}}{B}\right) + C$, where C is any arbitrary constant, then :

(a) $A = \frac{2}{3}$ (b) $B = a^{3/2}$ (c) $A = \frac{1}{3}$ (d) $B = a^{1/2}$

4. Let $\int x\sin x \cdot \sec^3 x\,dx = \frac{1}{2}(x \cdot f(x) - g(x)) + k$, then :

(a) $f(x) \notin (-1, 1)$
(b) $g(x) = \sin x$ has 6 solution for $x \in [-\pi, 2\pi]$
(c) $g'(x) = f(x), \forall x \in R$
(d) $f(x) = g(x)$ has no solution

5. If $\int (\sin 3\theta + \sin\theta)\cos\theta\, e^{\sin\theta} d\theta = (A\sin^3\theta + B\cos^2\theta + C\sin\theta + D\cos\theta + E)e^{\sin\theta} + F$, then :

(a) $A = -4$ (b) $B = -12$ (c) $C = -20$ (d) None of these

6. For $a > 0$, if $I = \int \sqrt{\frac{x}{a^3 - x^3}}\,dx = A\sin^{-1}\left(\frac{x^{3/2}}{B}\right) + C$, where C is any arbitrary constant, then :

(a) $A = \frac{2}{3}$ (b) $B = a^{3/2}$ (c) $A = \frac{1}{3}$ (d) $B = a^{1/2}$

7. If $f(\theta) = \lim_{n\to\infty}\sum_{r=0}^{n\theta}\frac{2r}{n\sqrt{(3\theta n - 2r)(n\theta + 2r)}}$ then :

(a) $f(1) = \frac{\pi}{6}$
(b) $f(\theta) = \frac{\theta}{2}\int_0^{\theta}\frac{dx}{\sqrt{\theta^2 - \left(x - \frac{\theta}{2}\right)^2}}$
(c) $f(\theta)$ is a constant function
(d) $y = f(\theta)$ is invertible

8. If $f(x+y) = f(x)f(y)$ for all x, y and $f(0) \neq 0$, and $F(x) = \frac{f(x)}{1+(f(x))^2}$ then :

(a) $\int_{-2010}^{2011} F(x)\,dx = \int_{0}^{2011} F(x)\,dx$
(b) $\int_{-2010}^{2011} F(x)\,dx - \int_{0}^{2010} F(x)\,dx = \int_{0}^{2011} F(x)\,dx$
(c) $\int_{-2010}^{2011} F(x)\,dx = 0$
(d) $\int_{-2010}^{2010} (2F(-x) - F(x))\,dx = 2\int_{0}^{2010} F(x)\,dx$

9. Let $J = \int_{-1}^{2} \left(\cot^{-1} \frac{1}{x} + \cot^{-1} x \right) dx, K = \int_{-2\pi}^{7\pi} \frac{\sin x}{|\sin x|} dx$. Then which of the following alternative(s) is/are correct ?

(a) $2J + 3K = 8\pi$ (b) $4J^2 + K^2 = 26\pi^2$ (c) $2J - K = 3\pi$ (d) $\frac{J}{K} = \frac{2}{5}$

10. Which of the following function(s) is/are even ?

(a) $f(x) = \int_0^x \ln\left(t + \sqrt{1+t^2}\right) dt$ (b) $g(x) = \int_0^x \frac{(2^t+1)t}{2^t - 1} dt$

(c) $h(x) = \int_0^x \left(\sqrt{1+t+t^2} - \sqrt{1-t+t^2}\right) dt$ (d) $l(x) = \int_0^x \ln\left(\frac{1-t}{1+t}\right) dt$

11. Let $l_1 = \lim_{x \to \infty} \sqrt{\frac{x - \cos^2 x}{x + \sin x}}$ and $l_2 = \lim_{h \to 0^+} \int_{-1}^{1} \frac{h dx}{h^2 + x^2}$. Then :

(a) Both l_1 and l_2 are less than 22/7

(b) One of the two limits is rational and other irrational

(c) $l_2 > l_1$

(d) l_2 is greater than 3 times of l_1

12. For $a > 0$, if $I = \int \sqrt{\frac{x}{a^3 - x^3}} dx = A \sin^{-1}\left(\frac{x^{3/2}}{B}\right) + C$, where C is any arbitrary constant, then :

(a) $A = \frac{2}{3}$ (b) $B = a^{3/2}$ (c) $A = \frac{1}{3}$ (d) $B = a^{1/2}$

13. If $\int \frac{dx}{1 - \sin^4 x} = a \tan x + b \tan^{-1}(c \tan x) + D$, then :

(a) $a = \frac{1}{2}$ (b) $b = \sqrt{2}$ (c) $c = \sqrt{2}$ (d) $b = \frac{1}{2\sqrt{2}}$

14. The value of definite integral :

$$\int_{-2014}^{2014} \frac{dx}{1 + \sin^{2015}(x) + \sqrt{1 + \sin^{4030}(x)}} \text{ equals :}$$

(a) 0 (b) 2014 (c) $(2014)^2$ (d) 4028

15. Let $L = \lim_{n \to \infty} \int_a^{\infty} \frac{n\, dx}{1 + n^2 x^2}$ where $a \in R$ then L can be :

(a) π (b) $\frac{\pi}{2}$ (c) 0 (d) $\frac{\pi}{3}$

16. Let $I = \int_0^1 \sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}}\, dx$ and $J = \int_0^1 \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\, dx$ then correct statement(s) is/are :

(a) $I + J = 2$ (b) $I - J = \pi$ (c) $I = \frac{2+\pi}{2}$ (d) $J = \frac{4-\pi}{2}$

Answers

1.	(b, c)	**2.**	(a, b, c, d)	**3.**	(a, b)	**4.**	(a, c, d)	**5.**	(a, b, c)	**6.**	(a, b)
7.	(a, b, d)	**8.**	(b, d)	**9.**	(a, b)	**10.**	(a, b, c, d)	**11.**	(a, b, c, d)	**12.**	(a, b)
13.	(a, c)	**14.**	(b)	**15.**	(a, b, c)	**16.**	(b, c)				

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

Let $f(x)=\int x^2\cos^2 x(2x+6\tan x-2x\tan^2 x)\,dx$ and $f(x)$ passes through the point $(\pi, 0)$

1. If $f:R-(2n+1)\frac{\pi}{2}\longrightarrow R$ then $f(x)$ be a :

(a) even function
(b) odd function
(c) neither even nor odd
(d) even as well as odd both

2. The number of solution(s) of the equation $f(x)=x^3$ in $[0, 2\pi]$ be :

(a) 0
(b) 3
(c) 2
(d) None of these

Paragraph for Question Nos. 3 to 4

Let $f(x)$ be a twice differentiable function defined on $(-\infty,\infty)$ such that $f(x)=f(2-x)$ and $f'\left(\frac{1}{2}\right)=f'\left(\frac{1}{4}\right)=0$. Then

3. The minimum number of values where $f''(x)$ vanishes on [0, 2] is :

(a) 2
(b) 3
(c) 4
(d) 5

4. $\int_{-1}^{1} f'(1+x)\,x^2 e^{x^2}\,dx$ is equal to :

(a) 1
(b) π
(c) 2
(d) 0

5. $\int_0^1 f(1-t)\,e^{-\cos\pi t}\,dt-\int_1^2 f(2-t)\,e^{\cos\pi t}\,dt$ is equal to :

(a) $\int_0^2 f'(t)\,e^{\cos\pi t}\,dt$
(b) 1
(c) 2
(d) π

Paragraph for Question Nos. 6 to 8

Consider the function $f(x)$ and $g(x)$, both defined from $R\to R$

$$f(x)=\frac{x^3}{2}+1-x\int_0^x g(t)\,dt \text{ and } g(x)=x-\int_0^1 f(t)\,dt, \text{ then}$$

6. Minimum value of $f(x)$ is :

(a) 0
(b) 1
(c) $\frac{3}{2}$
(d) Does not exist

7. The number of points of intersection of $f(x)$ and $g(x)$ is/are :

(a) 0 (b) 1 (c) 2 (d) 3

8. The area bounded by $g(x)$ with co-ordinate axes is (in square units) :

(a) $\frac{9}{4}$ (b) $\frac{9}{2}$ (c) $\frac{9}{8}$ (d) None of these

Paragraph for Question Nos. 9 to 11

Let $f(x)$ be function defined on $[0, 1]$ such that $f(1) = 0$ and for any $a \in (0, 1]$,

$\int_0^a f(x)\,dx - \int_a^1 f(x)\,dx = 2f(a) + 3a + b$ where b is constant.

9. $b =$

(a) $\frac{3}{2e} - 3$ (b) $\frac{3}{2e} - \frac{3}{2}$ (c) $\frac{3}{2e} + 3$ (d) $\frac{3}{2e} + \frac{3}{2}$

10. The length of the subtangent of the curve $y = f(x)$ at $x = 1/2$ is :

(a) $\sqrt{e} - 1$ (b) $\frac{\sqrt{e} - 1}{2}$ (c) $\sqrt{e} + 1$ (d) $\frac{\sqrt{e} + 1}{2}$

11. $\int_0^1 f(x)\,dx =$

(a) $\frac{1}{e}$ (b) $\frac{1}{2e}$ (c) $\frac{3}{2e}$ (d) $\frac{2}{e}$

Paragraph for Question Nos. 12 to 13

Let $f_0(x) = \ln x$ and for $n \geq 0$ and $x > 0$

Let $f_{n+1}(x) = \int_0^x f_n(t)\,dt$ then :

12. $f_3(x)$ equals :

(a) $\frac{x^3}{3}\left(\ln x - \frac{5}{6}\right)$ (b) $\frac{x^3}{3}\left(\ln x - \frac{11}{6}\right)$ (c) $\frac{x^3}{\lfloor 3}\left(\ln x - \frac{11}{6}\right)$ (d) $\frac{x^3}{\lfloor 3}\left(\ln x - \frac{5}{6}\right)$

13. Value of $\lim_{n \to \infty} \frac{(\lfloor n\,)\, f_n(1)}{\ln(n)}$:

(a) 0 (b) 1 (c) -1 (d) $-e$

Paragraph for Question Nos. 14 to 15

Let $f:R \to \left[\frac{3}{4}, \infty\right)$ be a surjective quadratic function with line of symmetry $2x - 1 = 0$ and $f(1) = 1$

14. If $g(x) = \dfrac{f(x) + f(-x)}{2}$ then $\displaystyle\int \frac{dx}{\sqrt{g(e^x) - 2}}$ is equal to :

(a) $\sec^{-1}(e^{-x}) + C$ (b) $\sec^{-1}(e^x) + C$ (c) $\sin^{-1}(e^{-x}) + C$ (d) $\sin^{-1}(e^x) + C$

(Where C is constant of integration)

15. $\displaystyle\int \frac{e^x}{f(e^x)} dx$

(a) $\cot^{-1}\left(\dfrac{2e^x - 1}{\sqrt{3}}\right) + C$ (b) $\dfrac{2}{\sqrt{3}} \cot^{-1}\left(\dfrac{2e^x + 1}{\sqrt{3}}\right) + C$

(c) $\tan^{-1}\left(\dfrac{2e^x + 1}{\sqrt{3}}\right) + C$ (d) $\dfrac{2}{\sqrt{3}} \tan^{-1}\left(\dfrac{2e^x - 1}{\sqrt{3}}\right) + C$

Paragraph for Question Nos. 16 to 17

Let $g(x) = x^C e^{Cx}$ and $f(x) = \displaystyle\int_0^x te^{2t}(1 + 3t^2)^{1/2} dt$. If $L = \displaystyle\lim_{x \to \infty} \frac{f'(x)}{g'(x)}$ is non-zero finite number then :

16. The value of C is :

(a) 7 (b) $\dfrac{3}{2}$ (c) 2 (d) 3

17. The value of L is :

(a) $\dfrac{2}{7}$ (b) $\dfrac{1}{2}$ (c) $\dfrac{\sqrt{3}}{4}$ (d) $\dfrac{\sqrt{3}}{2}$

Answers

1.	(a)	**2.**	(b)	**3.**	(c)	**4.**	(d)	**5.**	(a)	**6.**	(b)	**7.**	(a)	**8.**	(c)	**9.**	(a)	**10.**	(a)
11.	(c)	**12.**	(c)	**13.**	(c)	**14.**	(b)	**15.**	(d)	**16.**	(c)	**17.**	(d)						

Exercise-4 : Matching Type Problems

1.

	Column-I		Column-II
(A)	$\lim_{n\to\infty} 4\left[\frac{e^{\frac{1}{n}}}{n^2}+\frac{2}{n^2}e^{\frac{2}{n}}+\frac{3}{n^2}e^{\frac{3}{n}}+\ldots\frac{1}{n}e\right]=$	(P)	0
(B)	$\int_0^1 \ln\left(\frac{1}{x}-1\right)dx=$	(Q)	1
(C)	$\int_0^{10\pi}\left(\lim_{x\to y}\left(\frac{\sin x-\sin y}{x-y}\right)\right)dy=$	(R)	2
(D)	$\int_0^{\infty}\frac{\ln\left(x+\frac{1}{x}\right)dx}{(1+x^2)}=\frac{\pi}{2}\ln a$, then $a=$	(S)	4
		(T)	5

2. Match the following $\int f(x)\,dx$ is equal to, if

	Column-I		Column-II
(A)	$f(x)=\frac{1}{(x^2+1)\sqrt{x^2+2}}$	(P)	$\frac{x^5}{5(1-x^4)^{5/2}}+C$
(B)	$f(x)=\frac{1}{(x+2)\sqrt{x^2+6x+7}}$	(Q)	$\sin^{-1}\left(\frac{x+1}{(x+2)\sqrt{2}}\right)+C$
(C)	$f(x)=\frac{x^4+x^8}{(1-x^4)^{7/2}}$	(R)	$(\sqrt{x}-2)\sqrt{1-x}+\cos^{-1}\sqrt{x}+C$
(D)	$f(x)=\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}$	(S)	$-\tan^{-1}\sqrt{1+\frac{2}{x^2}}+C$
		(T)	$\frac{x^6}{6(1-x^4)^{5/2}}+C$

3.

	Column-I		Column-II
(A)	$\int_0^{\pi/2} \frac{\cos x}{(1+\sin x)(2+\sin x)} dx =$	(P)	$\frac{\pi}{6}$
(B)	$\int_0^{\frac{41\pi}{4}} \lvert \cos x \rvert dx =$	(Q)	$20+\frac{1}{\sqrt{2}}$
(C)	$\int_{-1/2}^{1/2} \left([x]+\ln\left(\frac{1+x}{1-x}\right)\right) dx =$ where [·] greatest integer function	(R)	$\ln 4 - \ln 3$
(D)	$\int_0^{\pi/2} \frac{2\sqrt{\cos\theta}}{3(\sqrt{\sin\theta}+\sqrt{\cos\theta})} d\theta =$	(S)	$-\frac{1}{2}$

4.

	Column-I		Column-II
(A)	If quardratic equation $3x^2+ax+1=0$ and $2x^2+bx+1=0$ have a common root then value of $5ab-2a^2-3b^2 =$	(P)	6
(B)	Number of solution of $x^4-2x^2\sin^2\frac{\pi x}{2}+1=0$ is/are	(Q)	1
(C)	Number of points of discontinuity $y=\frac{1}{u^2+u-2}$ where $u=\frac{1}{x-1}$ is/are	(R)	2
(D)	$\int \frac{dx}{\sqrt[3]{x^{5/2}(1+x)^{7/2}}} = A\left(\frac{x+1}{x}\right)^{-1/A} + C$ (Where C is integration constant), then $A =$	(S)	3

5. :

	Column-I		Column-II
(A)	$\int_0^{1.5} [x^2] dx$	(P)	$-\pi$
(B)	$\int_0^4 \{\sqrt{x}\}\, dx$ where $\{x\}$ denotes the fractional part of x	(Q)	$4(\sqrt{2}-1)$
(C)	$\int_0^{2\pi} [\sin x + \cos x]\, dx$	(R)	$\frac{7}{3}$
(D)	$\int_0^{\pi} \|\, \|\sin x\| - \|\cos x\| \,\|\, dx$	(S)	$2-\sqrt{2}$

Answers

1. A → S; B → P; C → P; D → S
2. A → S; B → Q; C → P; D → R
3. A → R; B → Q; C → S; D → P
4. A → Q; B → R; C → S; D → P
5. A → S; B → R; C → P; D → Q

Exercise-5 : Subjective Type Problems

1. $\int \frac{x+(\arccos 3x)^2}{\sqrt{1-9x^2}}\,dx = \frac{1}{k_1}\left(\sqrt{1-9x^2}+(\cos^{-1}3x)^{k_2}\right)+C$, then $k_1^2+k_2^2 =$

(where C is an arbitrary constant.)

2. If $\int_0^{\infty} \frac{x^3}{(a^2+x^2)^5}\,dx = \frac{1}{ka^6}$, then find the value of $\frac{k}{8}$.

3. Let $f(x) = x\cos x;\ x \in \left[\frac{3\pi}{2}, 2\pi\right]$ and $g(x)$ be its inverse. If $\int_0^{2\pi} g(x)\,dx = \alpha\pi^2+\beta\pi+\gamma$, where α, β and $\gamma \in R$, then find the value of $2(\alpha+\beta+\gamma)$.

4. If $\int (x^6+x^4+x^2)\sqrt{2x^4+3x^2+6}\,dx = \frac{(\alpha x^6+\beta x^4+\gamma x^2)^{3/2}}{18}+C$ where C is constant, then find the value of $(\beta+\gamma-\alpha)$.

5. If the value of the definite integral $\int_{-1}^{1} \cot^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)\cdot\left(\cot^{-1}\frac{x}{\sqrt{1-(x^2)^{|x|}}}\right)dx = \frac{\pi^2(\sqrt{a}-\sqrt{b})}{\sqrt{c}}$

where $a, b, c, \in N$ in their lowest from, then find the value of $(a+b+c)$.

6. The value of $\int \frac{\tan x}{\tan^2 x+\tan x+1}\,dx = x - \frac{2}{\sqrt{A}}\tan^{-1}\left(\frac{2\tan x+1}{\sqrt{A}}\right)+C$

Then the value of A is :

7. Let $\int_0^1 \frac{4x^3\,(1+(x^4)^{2010})}{(1+x^4)^{2012}}\,dx = \frac{\lambda}{\mu}$

where λ and μ are relatively prime positive integers. Find unit digit of μ.

8. Let $\int_1^{\sqrt{3}} \left(x^{2x^2+1} + \ln(x^{2x^{2x^2+1}})\right)dx = N$. Find the value of $(N-6)$.

9. If $\int \frac{dx}{\cos^3 x - \sin^3 x} = A\tan^{-1}(f(x)) + B\ln\left|\frac{\sqrt{2}+f(x)}{\sqrt{2}-f(x)}\right| + C$ where $f(x) = \sin x + \cos x$ find the value of $(12A + 9\sqrt{2}B) - 3$.

10. Find the value of $|a|$ for which the area of triangle included between the coordinate axes and any tangent to the curve $x^a y = \lambda^a$ is constant (where λ is constant.)

11. Let $I = \int_0^{\pi} x^6\,(\pi-x)^8\,dx$, then $\frac{\pi^{15}}{({}^{15}C_9)I} =$

12. If maximum value of $\int_0^1 (f(x))^3\,dx$ under the condition $-1 \le f(x) \le 1$; $\int_0^1 f(x)\,dx = 0$ is $\frac{p}{q}$ (where p and q are relatively prime positive integers.). Find $p+q$.

13. Let a differentiable function $f(x)$ satisfies $f(x)\cdot f'(-x) = f(-x)\cdot f'(x)$ and $f(0)=1$. Find the value of $\int_{-2}^{2} \frac{dx}{1+f(x)}$.

14. If $\{x\}$ denotes the fractional part of x, then $I = \int_0^{100} \{\sqrt{x}\}\,dx$, then the value of $\frac{9I}{155}$ is :

15. Let $I_n = \int_0^{\pi} \frac{\sin\left(n+\frac{1}{2}\right)x}{\sin\left(\frac{x}{2}\right)}dx$ where $n \in W$. If $I_1^2 + I_2^2 + I_3^2 + \ldots\ldots + I_{20}^2 = m\pi^2$, then find the largest prime factor of m.

16. If M be the maximum value of $72\int_0^y \sqrt{x^4+(y-y^2)^2}\,dx$ for $y \in [0,1]$, then find $\frac{M}{6}$.

17. Find the number of points where $f(\theta) = \int_{-1}^{1} \frac{\sin\theta\,dx}{1-2x\cos\theta+x^2}$ is discontinuous where $\theta \in [0, 2\pi]$.

18. Find the value of $\lim_{n\to\infty} \frac{1}{\sqrt{n}}\left(1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots\ldots+\frac{1}{\sqrt{n}}\right)$.

19. The maximum value of $\int_{-\pi/2}^{3\pi/2} \sin x\cdot f(x)\,dx$, subject to the condition $|f(x)| \le 5$ is M, then $\frac{M}{10}$ is equal to :

20. Given a function g, continuous everywhere such that $g(1)=5$ and $\int_0^1 g(t)\,dt = 2$. If $f(x) = \frac{1}{2}\int_0^x (x-t)^2 g(t)\,dt$, then find the value of $f'''(1)+f''(1)$.

21. If $f(n) = \frac{1}{\pi}\int_0^{\pi/2} \frac{\sin^2(n\theta)\,d\theta}{\sin^2\theta}$, $n \in N$, then evaluate $\frac{f(15)+f(3)}{f(12)-f(10)}$.

22. Let $f(2-x) = f(2+x)$ and $f(4-x) = f(4+x)$. Function $f(x)$ satisfies $\int_0^2 f(x)\,dx = 5$. If $\int_0^{50} f(x)\,dx = I$. Find $[\sqrt{I}-3]$. (where $[\cdot]$ denotes greatest integer function.)

23. Let $I_n = \int_{-1}^{1} |x|\left(1 + x + \frac{x^2}{2} + \frac{x^3}{3} + ... + \frac{x^{2n}}{2n}\right)dx$. If $\lim_{n\to\infty} I_n$ can be expressed as rational $\frac{p}{q}$ in its lowest form, then find the value of $\frac{pq(p+q)}{10}$.

24. Let $\lim_{n\to\infty} n^{-\frac{1}{2}\left(1+\frac{1}{n}\right)} \cdot (1^1 \cdot 2^2 \cdot 3^3 \cdot ...... n^n)^{\frac{1}{n^2}} = e^{\frac{-p}{q}}$

where p and q are relative prime positive integers. Find the value of $|p+q|$.

25. If $\int_a^b |\sin x|\, dx = 8$ and $\int_0^{a+b} |\cos x|\, dx = 9$ then the value of $\frac{1}{\sqrt{2}x}\left|\int_a^b x \sin x\, dx\right|$ is :

26. If $f(x)$, $g(x)$, $h(x)$ and $\phi(x)$ are polynomial in x,

$$\left(\int_1^x f(x)\, h(x)\, dx\right)\left(\int_1^x g(x)\, \phi(x)\, dx\right) - \left(\int_1^x f(x)\, \phi(x)\, dx\right)\left(\int_1^x g(x)\, h(x)\, dx\right)$$

is divisible by $(x-1)^\lambda$. Find maximum value of λ.

27. If $\int_0^2 (3x^2 - 3x + 1)\cos(x^3 - 3x^2 + 4x - 2)dx = a\sin(b)$, where a and b are positive integers. Find the value of $(a+b)$.

28. let $f(x) = \int_0^x e^{x-y} f'(y)\, dy - (x^2 - x + 1)e^x$

Find the number of roots of the equation $f(x) = 0$.

29. For a positive integer n, let $I_n = \int_{-\pi}^{\pi} \left(\frac{\pi}{2} - |x|\right)\cos nx\, dx$

Find the value of $[I_1 + I_2 + I_3 + I_4]$ where [·] denotes greatest integer function.

Answers

1.	90	**2.**	3	**3.**	3	**4.**	7	**5.**	7	**6.**	3	**7.**	1
8.	7	**9.**	8	**10.**	1	**11.**	9	**12.**	5	**13.**	2	**14.**	3
15.	5	**16.**	4	**17.**	3	**18.**	2	**19.**	2	**20.**	7	**21.**	9
22.	8	**23.**	3	**24.**	5	**25.**	2	**26.**	4	**27.**	2	**28.**	1
29.	4												

Chapter 6 – Area Under Curves

6

AREA UNDER CURVES

Exercise-1 : Single Choice Problems

1. The area enclosed by the curve
$[x+3y]=[x-2]$ where $x \in [3, 4)$ is :
(where [·] denotes greatest integer function.)

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) 1

2. The area of region enclosed by the curves $y=x^2$ and $y=\sqrt{|x|}$ is :

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{16}{3}$

3. Area enclosed by the figure described by the equation $x^4+1=2x^2+y^2$, is :

(a) 2 (b) $\frac{16}{3}$ (c) $\frac{8}{3}$ (d) $\frac{4}{3}$

4. The area defined by $|y| \le e^{-|x|}-\frac{1}{2}$ in cartesian co-ordinate system, is :

(a) $(4-2\ln 2)$ (b) $(4-\ln 2)$ (c) $(2-\ln 2)$ (d) $(2-2\ln 2)$

5. For each positive integer $n>1$; A_n represents the area of the region restricted to the following two inequalities : $\frac{x^2}{n^2}+y^2 \le 1$ and $x^2+\frac{y^2}{n^2} \le 1$. Find $\lim_{n\to\infty} A_n$.

(a) 4 (b) 1 (c) 2 (d) 3

6. The ratio in which the area bounded by curves $y^2=12x$ and $x^2=12y$ is divided by the line $x=3$ is :

(a) 7 : 15 (b) 15 : 49 (c) 1 : 3 (d) 17 : 49

7. The value of positive real parameter 'a' such that area of region bounded by parabolas $y=x-ax^2$, $ay=x^2$ attains its maximum value is equal to :

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{3}$ (d) 1

8. For $0 < r < 1$, let n_r denotes the line that is normal to the curve $y = x^r$ at the point $(1, 1)$. Let S_r denotes the region in the first quadrant bounded by the curve $y = x^r$; the x-axis and the line n_r. Then the value of r that minimizes the area of S_r is :

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}-1$ (c) $\frac{\sqrt{2}-1}{2}$ (d) $\sqrt{2}-\frac{1}{2}$

9. The area bounded by $|x| = 1 - y^2$ and $|x| + |y| = 1$ is :

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) 1

10. Point A lies on curve $y = e^{-x^2}$ and has the coordinate (x, e^{-x^2}) where $x > 0$. Point B has coordinates $(x, 0)$. If 'O' is the origin, then the maximum area of ΔAOB is :

(a) $\frac{1}{\sqrt{8e}}$ (b) $\frac{1}{\sqrt{4e}}$ (c) $\frac{1}{\sqrt{2e}}$ (d) $\frac{1}{\sqrt{e}}$

11. The area enclosed between the curves $y = ax^2$ and $x = ay^2$ $(a > 0)$ is 1 sq. unit, then the value of a is :

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{2}$ (c) 1 (d) $\frac{1}{3}$

12. Let $f(x) = x^3 - 3x^2 + 3x + 1$ and g be the inverse of it ; then area bounded by the curve $y = g(x)$ with x-axis between $x = 1$ to $x = 2$ is (in square units) :

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) 1

13. Area bounded by $x^2y^2 + y^4 - x^2 - 5y^2 + 4 = 0$ is equal to :

(a) $\frac{4\pi}{3} + \sqrt{2}$ (b) $\frac{4\pi}{3} - \sqrt{2}$ (c) $\frac{4\pi}{3} + 2\sqrt{3}$ (d) none of these

14. Let $f : R^+ \to R^+$ is an invertible function such that $f'(x) > 0$ and $f''(x) > 0 \ \forall \ x \in [1, 5]$. If $f(1) = 1$ and $f(5) = 5$ and area bounded by $y = f(x)$, x-axis, $x = 1$ and $x = 5$ is 8 sq. units. Then the area bounded by $y = f^{-1}(x)$, x-axis, $x = 1$ and $x = 5$ is :

(a) 12 (b) 16 (c) 18 (d) 20

15. A circle centered at origin and having radius π units is divided by the curve $y = \sin x$ in two parts. Then area of the upper part equals to :

(a) $\frac{\pi^2}{2}$ (b) $\frac{\pi^3}{4}$ (c) $\frac{\pi^3}{2}$ (d) $\frac{\pi^3}{8}$

16. The area of the loop formed by $y^2 = x(1 - x^3)dx$ is :

(a) $\int_0^1 \sqrt{x - x^4}\, dx$ (b) $2\int_0^1 \sqrt{x - x^4}\, dx$

(c) $\int_{-1}^1 \sqrt{x - x^4}\, dx$ (d) $4\int_0^{1/2} \sqrt{x - x^4}\, dx$

17. If $f(x) = \min\left[x^2, \sin\frac{x}{2}, (x-2\pi)^2\right]$, the area bounded by the curve $y = f(x)$, x-axis, $x = 0$ and $x = 2\pi$ is given by

(**Note :** x_1 is the point of intersection of the curves x^2 and $\sin\frac{x}{2}$; x_2 is the point of intersection of the curves $\sin\frac{x}{2}$ and $(x-2\pi)^2$)

(a) $\int_0^{x_1}\left(\sin\frac{x}{2}\right)dx + \int_{x_1}^{\pi} x^2 dx + \int_{\pi}^{x_2}(x-2\pi)^2 dx + \int_{x_2}^{2\pi}\left(\sin\frac{x}{2}\right)dx$

(b) $\int_0^{x_1} x^2 dx + \int_{x_1}^{x_2}\left(\sin\frac{x}{2}\right)dx + \int_{x_2}^{2\pi}(x-2\pi)^2 dx$, where $x_1 \in \left(0, \frac{\pi}{3}\right)$ and $x_2 \in \left(\frac{5\pi}{3}, 2\pi\right)$

(c) $\int_0^{x_1} x^2 dx + \int_{x_1}^{x_2}\sin\left(\frac{x}{2}\right)dx + \int_{x_2}^{2\pi}(x-2\pi)^2 dx$, where $x_1 \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$ and $x_2 \in \left(\frac{3\pi}{2}, 2\pi\right)$

(d) $\int_0^{x_1} x^2 dx + \int_{x_1}^{x_2}\sin\left(\frac{x}{2}\right)dx + \int_{x_2}^{2\pi}(x-2\pi)^2 dx$, where $x_1 \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and $x_2 \in (\pi, 2\pi)$

18. The area enclosed between the curves $|x|+|y| \geq 2$ and $y^2 = 4\left(1-\frac{x^2}{9}\right)$ is :

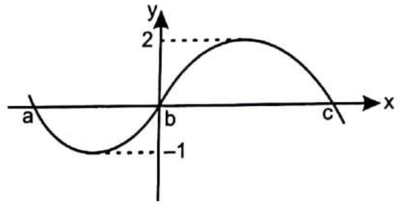
(a) $(6\pi-4)$ sq. units (b) $(6\pi-8)$ sq. units (c) $(3\pi-4)$ sq. units (d) $(3\pi-2)$ sq. units

Answers

1.	(b)	**2.**	(b)	**3.**	(c)	**4.**	(d)	**5.**	(a)	**6.**	(b)	**7.**	(d)	**8.**	(b)	**9.**	(c)	**10.**	(a)
11.	(d)	**12.**	(b)	**13.**	(c)	**14.**	(b)	**15.**	(c)	**16.**	(b)	**17.**	(b)	**18.**	(b)				

Exercise-2 : One or More than One Answer is/are Correct

1. Let $f(x)$ be a polynomial function of degree 3 where $a < b < c$ and $f(a) = f(b) = f(c)$. If the graph of $f(x)$ is as shown, which of the following statements are **INCORRECT** ? (Where $c > |a|$)



(a) $\int_a^c f(x)\,dx = \int_b^c f(x)\,dx + \int_a^b f(x)\,dx$

(b) $\int_a^c f(x)\,dx < 0$

(c) $\int_a^b f(x)\,dx < \int_c^b f(x)\,dx$

(d) $\frac{1}{b-a}\int_a^b f(x)\,dx > \frac{1}{c-b}\int_b^c f(x)\,dx$

2. $T_n = \sum_{r=2n}^{3n-1} \frac{r}{r^2+n^2}$, $S_n = \sum_{r=2n+1}^{3n} \frac{r}{r^2+n^2}$, then $\forall\, n \in \{1, 2, 3, \ldots\}$:

(a) $T_n > \frac{1}{2}\ln 2$ (b) $S_n < \frac{1}{2}\ln 2$ (c) $T_n < \frac{1}{2}\ln 2$ (d) $S_n > \frac{1}{2}\ln 2$

3. If a curve $y = a\sqrt{x} + bx$ passes through point (1, 2) and the area bounded by curve, line $x = 4$ and x-axis is 8, then :

(a) $a = 3$ (b) $b = 3$ (c) $a = -1$ (d) $b = -1$

4. Area enclosed by the curves $y = x^2 + 1$ and a normal drawn to it with gradient -1; is equal to :

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

Answers

1.	(b, c, d)	**2.**	(a, b)	**3.**	(a, d)	**4.**	(d)				

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 3

Let $f : A \to B$ $f(x) = \dfrac{x+a}{bx^2+cx+2}$, where A represent domain set and B represent range set of function $f(x)$, $a, b, c \in R$, $f(-1) = 0$ and $y = 1$ is an asymptote of $y = f(x)$ and $y = g(x)$ is the inverse of $f(x)$.

1. $g(0)$ is equal to :

(a) -1 (b) -3 (c) $-\dfrac{5}{2}$ (d) $-\dfrac{3}{2}$

2. Area bounded between the curves $y = f(x)$ and $y = g(x)$ is :

(a) $2\sqrt{5} + \ln\left(\dfrac{3-\sqrt{5}}{3+\sqrt{5}}\right)$ (b) $3\sqrt{5} + 2\ln\left(\dfrac{3+\sqrt{5}}{3-\sqrt{5}}\right)$

(c) $3\sqrt{5} + 4\ln\left(\dfrac{3-\sqrt{5}}{3+\sqrt{5}}\right)$ (d) $3\sqrt{5} + 2\ln\left(\dfrac{3-\sqrt{5}}{3+\sqrt{5}}\right)$

3. Area of region enclosed by asymptotes of curves $y = f(x)$ and $y = g(x)$ is :

(a) 4 (b) 9 (c) 12 (d) 25

Paragraph for Question Nos. 4 to 6

For $j = 0, 1, 2, \ldots n$ let S_j be the area of region bounded by the x-axis and the curve $ye^x = \sin x$ for $j\pi \le x \le (j+1)\pi$

4. The value of S_0 is :

(a) $\dfrac{1}{2}(1+e^{\pi})$ (b) $\dfrac{1}{2}(1+e^{-\pi})$ (c) $\dfrac{1}{2}(1-e^{-\pi})$ (d) $\dfrac{1}{2}(e^{\pi}-1)$

5. The ratio $\dfrac{S_{2009}}{S_{2010}}$ equals :

(a) $e^{-\pi}$ (b) e^{π} (c) $\dfrac{1}{2}e^{\pi}$ (d) $2e^{\pi}$

6. The value of $\sum_{j=0}^{\infty} S_j$ equals to :

(a) $\dfrac{e^{\pi}(1+e^{\pi})}{2(e^{\pi}-1)}$ (b) $\dfrac{1+e^{\pi}}{2(e^{\pi}-1)}$ (c) $\dfrac{1+e^{\pi}}{e^{\pi}-1}$ (d) $\dfrac{e^{\pi}(1+e^{\pi})}{(e^{\pi}-1)}$

Answers

1.	(a)	**2.**	(d)	**3.**	(b)	**4.**	(b)	**5.**	(b)	**6.**	(b)

Exercise-4 : Matching Type Problems

1.

	Column-I		Column-II
(A)	Area of region formed by points (x, y) satisfying $[x]^2 = [y]^2$ for $0 \le x \le 4$ is equal to (where [ ] denotes greatest integer function)	**(P)**	48
(B)	The area of region formed by points (x, y) satisfying $x + y \le 6$, $x^2 + y^2 \le 6y$ and $y^2 \le 8x$ is $\frac{k\pi - 2}{12}$, then $k =$	**(Q)**	27
(C)	The area in the first quardant bounded by the curve $y = \sin x$ and the line $\frac{2y-1}{\sqrt{2}-1} = \frac{2}{\pi}(6x - \pi)$ is $\left[\frac{\sqrt{3}-\sqrt{2}}{2} - \frac{(\sqrt{2}+1)\pi}{k}\right]$, then $k =$	**(R)**	7
(D)	If the area bounded by the graph of $y = xe^{-ax}$ $(a > 0)$ and the abscissa axis is $\frac{1}{9}$ then the value of 'a' is equal to	**(S)**	4
		(T)	3

Answers

1. A → R ; B → Q ; C → P ; D → T

Exercise-5 : Subjective Type Problems

1. Let f be a differentiable function satisfying the condition $f\left(\frac{x}{y}\right)=\frac{f(x)}{f(y)}(y \neq 0, f(y) \neq 0)$ $\forall x, y \in R$ and $f'(1)=2$. If the smaller area enclosed by $y=f(x)$, $x^2+y^2=2$ is A, then find $[A]$, where $[\cdot]$ represents the greatest integer function.

2. Let $f(x)$ be a function which satisfy the equation $f(xy)=f(x)+f(y)$ for all $x>0, y>0$ such that $f'(1)=2$. Let A be the area of the region bounded by the curves $y=f(x)$, $y=|x^3-6x^2+11x-6|$ and $x=0$, then find value of $\frac{28}{17}A$.

3. If the area bounded by circle $x^2+y^2=4$, the parabola $y=x^2+x+1$ and the curve $y=\left[\sin^2\frac{x}{4}+\cos\frac{x}{4}\right]$, (where [ ] denotes the greates integer function) and x-axis is $\left(\sqrt{3}+\frac{2\pi}{3}-\frac{1}{k}\right)$, then the numerical quantity k should be :

4. Let the function $f:[-4,4]\to[-1,1]$ be defined implicitly by the equation $x+5y-y^5=0$. If the area of triangle formed by tangent and normal to $f(x)$ at $x=0$ and the line $y=5$ is A, find $\frac{A}{13}$.

5. Area of the region bounded by $[x]^2=[y]^2$, if $x\in[1,5]$, where [ ] denotes the greatest integer function, is :

6. Consider $y=x^2$ and $f(x)$ where $f(x)$, is a differentiable function satisfying $f(x+1)+f(z-1)=f(x+z)\ \forall x, z \in R$ and $f(0)=0$; $f'(0)=4$. If area bounded by curve $y=x^2$ and $y=f(x)$ is Δ, find the value of $\left(\frac{3}{16}\Delta\right)$.

7. The least integer which is greater than or equal to the area of region in $x-y$ plane satisfying $x^6-x^2+y^2\le 0$ is :

8. The set of points (x, y) in the plane statisfying $x^{2/5}+|y|=1$ form a curve enclosing a region of area $\frac{p}{q}$ square units, where p and q are relatively prime positive integers. Find $p-q$.

Answers

1.	1	2.	7	3.	6	4.	5	5.	8	6.	2	7.	2
8.	1												

□□□

Chapter 7 – Differential Equations

7

DIFFERENTIAL EQUATIONS

Exercise-1 : Single Choice Problems

1. $\frac{dy}{dx}\left(\frac{1+\cos x}{y}\right)=-\sin x$ and $f\left(\frac{\pi}{2}\right)=-1$, then $f(0)$ is :

(a) 2 (b) 1 (c) 3 (d) 4

2. The differential equation satisfied by family of curves $y=Ae^{x}+Be^{3x}+Ce^{5x}$ where A, B, C are arbitrary constants is :

(a) $\frac{d^3y}{dx^3}-9\frac{d^2y}{dx^2}+23\frac{dy}{dx}+15y=0$

(b) $\frac{d^3y}{dx^3}+9\frac{d^2y}{dx^2}-23\frac{dy}{dx}-15y=0$

(c) $\frac{d^3y}{dx^3}+9\frac{d^2y}{dx^2}-23\frac{dy}{dx}+15y=0$

(d) $\frac{d^3y}{dx^3}-9\frac{d^2y}{dx^2}+23\frac{dy}{dx}-15y=0$

3. If $y=y(x)$ and it follows the relation $e^{xy^2}+y\cos(x^2)=5$ then $y'(0)$ is equal to :

(a) 4 (b) −16 (c) −4 (d) 16

4. $(x^2+y^2)dy=xydx$. If $y(x_0)=e$, $y(1)=1$, then the value of x_0 is equal to :

(a) $\sqrt{3}\,e$ (b) $\sqrt{e^2-\frac{1}{2}}$ (c) $\sqrt{\frac{e^2-1}{2}}$ (d) $\sqrt{e^2+\frac{1}{2}}$

5. The differential equation $\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{y}$ determines a family of circles with :

(a) Variable radii and fixed centre at $(0, 1)$

(b) Variable radii and fixed centre at $(0, -1)$

(c) Fixed radius 1 and variable centres along x-axis

(d) Fixed radius 1 and variable centres along y-axis

6. Interval contained in the domain of definition of non-zero solutions of the differential equation $(x-3)^2y'+y=0$ is :

(a) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ (b) $\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$ (c) $\left(\frac{\pi}{8},\frac{5\pi}{4}\right)$ (d) $(-\pi, \pi)$

7. A function $y = f(x)$ satisfies the differential equation $(x+1)f'(x) - 2(x^2+x)f(x) = \dfrac{e^{x^2}}{(x+1)}$; $\forall\, x > -1$. If $f(0) = 5$, then $f(x)$ is :

(a) $\left(\dfrac{3x+5}{x+1}\right)\cdot e^{x^2}$ (b) $\left(\dfrac{6x+5}{x+1}\right)\cdot e^{x^2}$

(c) $\left(\dfrac{6x+5}{(x+1)^2}\right)\cdot e^{x^2}$ (d) $\left(\dfrac{5-6x}{x+1}\right)\cdot e^{x^2}$

8. The solution of the differential equation $2x^2y\dfrac{dy}{dx} = \tan(x^2y^2) - 2xy^2$ given $y(1) = \sqrt{\dfrac{\pi}{2}}$ is :

(a) $\sin(x^2y^2) - 1 = 0$ (b) $\cos\left(\dfrac{\pi}{2} + x^2y^2\right) + x = 0$

(c) $\sin(x^2y^2) = e^{x-1}$ (d) $\sin(x^2y^2) = e^{2(x-1)}$

9. The differential equation whose general solution is given by $y = C_1\cos(x + C_2) - C_3e^{-x+C_4} + C_5\sin x$, where $C_1, C_2, \ldots\ldots, C_5$ are constants is :

(a) $\dfrac{d^4y}{dx^4} - \dfrac{d^2y}{dx^2} + y = 0$ (b) $\dfrac{d^3y}{dx^3} - \dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - y = 0$

(c) $\dfrac{d^3y}{dx^3} + \dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} + y = 0$ (d) $\dfrac{d^3y}{dx^3} + \dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = 0$

10. If $y = e^{(\alpha+1)x}$ be solution of differential equation $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4y = 0$; then α is :

(a) 0 (b) 1 (c) −1 (d) 2

11. The order and degree of the differential equation $\left(\dfrac{dy}{dx}\right)^{1/3} - 4\dfrac{d^2y}{dx^2} - 7x = 0$ are α and β, then the value of $(\alpha + \beta)$ is :

(a) 3 (b) 4 (c) 2 (d) 5

12. General solution of differential equation of $f(x)\dfrac{dy}{dx} = f^2(x) + f(x)y + f'(x)y$ is :

(c being arbitary constant.)

(a) $y = f(x) + ce^x$ (b) $y = -f(x) + ce^x$

(c) $y = -f(x) + ce^x f(x)$ (d) $y = c\,f(x) + e^x$

13. The order and degree respectively of the differential equation of all tangent lines to parabola $x^2 = 2y$ is :

(a) 1, 2 (b) 2, 1

(c) 1, 1 (d) 1, 3

14. The general solution of the differential equation $\frac{dy}{dx}+x(x+y)=x(x+y)^3-1$ is :

(a) $\ln\left|\frac{(x+y+1)(x+y-1)}{(x+y)^4}\right|=x^2+C$

(b) $\ln\left|\frac{(x+y+1)(x+y-1)}{(x+y)^2}\right|=x^2+C$

(c) $2\ln\left|\frac{(x+y+1)(x+y-1)}{(x+y)^2}\right|=x^2+C$

(d) $\ln\left|\frac{(x+y+1)(x+y-1)}{(x+y)^2}\right|=x+C$

(where C is arbitrary constant.)

15. The general solution of $\frac{dy}{dx}=2y\tan x+\tan^2 x$ is :

(a) $y\cos^2 x=\frac{x}{2}-\frac{\sin 2x}{4}+C$

(b) $y\sec^2 x=\frac{x}{2}-\frac{\sin 2x}{4}+C$

(c) $y\cos^2 x=\frac{x}{2}-\frac{\cos 2x}{4}+C$

(d) $y\cos^2 x=\frac{x}{2}-\frac{\sin 2x}{2}+C$

(where C is an arbitrary constant.)

16. The solution of differential equation $\frac{d^2y}{dx^2}=\frac{dy}{dx}, y(0)=3$ and $y'(0)=2$:

(a) is a periodic function

(b) approaches to zero as $x\to-\infty$

(c) has an asymptote parallel to x-axis

(d) has an asymptote parallel to y-axis

17. The solution of the differential equation $(x^2+1)\frac{d^2y}{dx^2}=2x\left(\frac{dy}{dx}\right)$ under the conditions $y(0)=1$ and $y'(0)=3$, is :

(a) $y=x^2+3x+1$

(b) $y=x^3+3x+1$

(c) $y=x^4+3x+1$

(d) $y=3\tan^{-1}x+x^2+1$

18. The differential equation of the family of curves $cy^2=2x+c$ (where c is an arbitrary constant.) is :

(a) $\frac{xdy}{dx}=1$

(b) $\left(\frac{dy}{dx}\right)^2=\frac{2xdy}{dx}+1$

(c) $y^2=2xy\frac{dy}{dx}+1$

(d) $y^2=\frac{2ydy}{dx}+1$

19. The solution of the equation $\frac{dy}{dx}+\frac{1}{x}\tan y=\frac{1}{x^2}\tan y\sin y$ is :

(a) $2y=\sin y(1-2cx^2)$

(b) $2x=\cot y(1+2cx^2)$

(c) $2x=\sin y(1-2cx^2)$

(d) $2x\sin y=1-2cx^2$

20. Solution of the differential equation $xdy-ydx-\sqrt{x^2+y^2}dx=0$ is :

(a) $y-\sqrt{x^2+y^2}=cx^2$

(b) $y+\sqrt{x^2+y^2}=cx$

(c) $x-\sqrt{x^2+y^2}=cx^2$

(d) $y+\sqrt{x^2+y^2}=cx^2$

21. Let $f(x)$ be differentiable function on the interval $(0, \infty)$ such that $f(1) = 1$ and $\lim_{t \to x}\left(\frac{t^3 f(x) - x^3 f(t)}{t^2 - x^2}\right) = \frac{1}{2} \; \forall \, x > 0$, then $f(x)$ is :

(a) $\frac{1}{4x} + \frac{3x^2}{4}$ (b) $\frac{3}{4x} + \frac{x^3}{4}$ (c) $\frac{1}{4x} + \frac{3x^3}{4}$ (d) $\frac{1}{4x^3} + \frac{3x}{4}$

22. The population $p(t)$ at time 't' of a certain mouse species satisfies the differential equation $\frac{d}{dt} p(t) = 0.5\, p(t) - 450$. If $p(0) = 850$, then the time at which the population becomes zero is :

(a) $\frac{1}{2} \ln 18$ (b) $\ln 18$ (c) $2 \ln 18$ (d) $\ln 9$

23. The solution of the differential equation $\sin 2y \frac{dy}{dx} + 2 \tan x \cos^2 y = 2 \sec x \cos^3 y$ is :

(where C is arbitrary constant)

(a) $\cos y \sec x = \tan x + C$ (b) $\sec y \cos x = \tan x + C$

(c) $\sec y \sec x = \tan x + C$ (d) $\tan y \sec x = \sec x + C$

24. The solution of the differential equation $\frac{dy}{dx} = (4x + y + 1)^2$ is :

(where C is arbitrary constant)

(a) $4x + y + 1 = 2 \tan(2x + y + C)$ (b) $4x + y + 1 = 2 \tan(x + 2y + C)$

(c) $4x + y + 1 = 2 \tan(2y + C)$ (d) $4x + y + 1 = 2 \tan(2x + C)$

25. If a curve is such that line joining origin to any point $P(x, y)$ on the curve and the line parallel to y-axis through P are equally inclined to tangent to curve at P, then the differential equation of the curve is :

(a) $x\left(\frac{dy}{dx}\right)^2 - 2y \frac{dy}{dx} = x$ (b) $x\left(\frac{dy}{dx}\right)^2 + 2y \frac{dy}{dx} = x$

(c) $y\left(\frac{dy}{dx}\right)^2 - 2x \frac{dy}{dx} = x$ (d) $y\left(\frac{dy}{dx}\right)^2 - 2y \frac{dy}{dx} = x$

26. If $y = f(x)$ satisfy the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$; $f(1) = 1$; then value of $f(3)$ equals :

(a) 7 (b) 5 (c) 9 (d) 27

27. Let $y = f(x)$ and $\frac{x}{y}\frac{dy}{dx} = \frac{3x^2 - y}{2y - x^2}$; $f(1) = 1$ then the possible value of $\frac{1}{3} f(3)$ equals :

(a) 9 (b) 4 (c) 3 (d) 2

Answers

1.	(a)	**2.**	(d)	**3.**	(b)	**4.**	(a)	**5.**	(c)	**6.**	(a)	**7.**	(b)	**8.**	(c)	**9.**	(c)	**10.**	(b)
11.	(d)	**12.**	(c)	**13.**	(a)	**14.**	(b)	**15.**	(a)	**16.**	(c)	**17.**	(b)	**18.**	(c)	**19.**	(c)	**20.**	(d)
21.	(c)	**22.**	(c)	**23.**	(c)	**24.**	(d)	**25.**	(a)	**26.**	(a)	**27.**	(c)						

Exercise-2 : One or More than One Answer is/are Correct

1. Let $y = f(x)$ be a real valued function satisfying $x\frac{dy}{dx} = x^2 + y - 2$, $f(1) = 1$, then :
   (a) $f(x)$ is minimum at $x = 1$
   (b) $f(x)$ is maximum at $x = 1$
   (c) $f(3) = 5$
   (d) $f(2) = 3$

2. Solution of differential equation $x\cos x\left(\frac{dy}{dx}\right) + y(x\sin x + \cos x) = 1$ is :
   (a) $xy = \sin x + c\cos x$
   (b) $xy\sec x = \tan x + c$
   (c) $xy + \sin x + c\cos x = 0$
   (d) None of these
   (where C is an arbitrary constant.)

3. If a differentiable function satisfies $(x-y)f(x+y) - (x+y)f(x-y) = 2(x^2y - y^3)\ \forall\ x, y \in R$ and $f(1) = 2$, then :
   (a) $f(x)$ must be polynomial function
   (b) $f(3) = 12$
   (c) $f(0) = 0$
   (d) $f(3) = 13$

4. A function $y = f(x)$ satisfies the differential equation
$$f(x)\sin 2x - \cos x + (1 + \sin^2 x)\, f'(x) = 0$$
   with $f(0) = 0$. The value of $f\left(\frac{\pi}{6}\right)$ equals to :
   (a) $\frac{2}{5}$
   (b) $\frac{3}{5}$
   (c) $\frac{1}{5}$
   (d) $\frac{4}{5}$

5. Solution of the differential equation $(2 + 2x^2\sqrt{y})\,y\,dx + (x^2\sqrt{y} + 2)x\,dy = 0$ is/are :
   (a) $xy(x^2\sqrt{y} + 5) = c$
   (b) $xy(x^2\sqrt{y} + 3) = c$
   (c) $xy(y^2\sqrt{x} + 3) = c$
   (d) $xy(y^2\sqrt{x} + 5) = c$

6. If $y(x)$ satisfies the differential equation $\frac{dy}{dx} = \sin 2x + 3y\cot x$ and $y\left(\frac{\pi}{2}\right) = 2$ then which of the following statement(s) is/are correct ?
   (a) $y\left(\frac{\pi}{6}\right) = 0$
   (b) $y'\left(\frac{\pi}{3}\right) = \frac{9 - 3\sqrt{2}}{2}$
   (c) $y(x)$ increases in the interval
   (d) $\int_{-\pi/2}^{\pi/2} y(x)\,dx = x$

Answers

1.	(a, c)	2.	(a, b)	3.	(a, b, c)	4.	(a)	5.	(b)	6.	(a, c)

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

A differentiable function $y = g(x)$ satisfies $\int_0^x (x-t+1)g(t)\,dt = x^4 + x^2; \forall x \geq 0$.

1. $y = g(x)$ satisfies the differential equation :

(a) $\frac{dy}{dx} - y = 12x^2 + 2$ (b) $\frac{dy}{dx} + 2y = 12x^2 + 2$

(c) $\frac{dy}{dx} + y = 12x^2 + 2$ (d) $\frac{dy}{dx} + y = 12x + 2$

2. The value of $g(0)$ equals to :

(a) 0 (b) 1 (c) e^2 (d) Data insufficient

Paragraph for Question Nos. 3 to 5

Suppose f and g are differentiable functions such that $xg(f(x))f'(g(x))g'(x) = f(g(x))$ $g'(f(x))f'(x)\ \forall x \in R$ and f is positive, g is positive $\forall x \in R$. Also $\int_0^x f(g(t))\,dt = \frac{1}{2}(1-e^{-2x})$ $\forall x \in R, g(f(0)) = 1$ and $h(x) = \frac{g(f(x))}{f(g(x))}\ \forall x \in R$.

3. The graph of $y = h(x)$ is symmetric with respect to line :

(a) $x = -1$ (b) $x = 0$ (c) $x = 1$ (d) $x = 2$

4. The value of $f(g(0)) + g(f(0))$ is equal to :

(a) 1 (b) 2 (c) 3 (d) 4

5. The largest possible value of $h(x)\ \forall x \in R$ is :

(a) 1 (b) $e^{1/3}$ (c) e (d) e^2

Paragraph for Question Nos. 6 to 8

Given a function 'g' which has a derivative $g'(x)$ for every real x and which satisfy $g'(0) = 2$ and $g(x+y) = e^y g(x) + e^x g(y)$ for all x and y.

6. The function $g(x)$ is :

(a) $x(2 + xe^x)$ (b) $x(e^x + 1)$ (c) $2xe^x$ (d) $x + \ln(x+1)$

7. The range of function $g(x)$ is :

(a) R (b) $\left[-\frac{2}{e}, \infty\right)$ (c) $\left[-\frac{1}{e}, \infty\right)$ (d) $[0, \infty)$

8. The value of $\lim\limits_{x \to -\infty} g(x)$ is :

(a) 0 (b) 1 (c) 2 (d) Does not exist

Answers

1.	(c)	**2.**	(a)	**3.**	(c)	**4.**	(b)	**5.**	(c)	**6.**	(c)	**7.**	(b)	**8.**	(a)

Exercise-4 : Matching Type Problems

1.

	Column-I (Differential equation)		Column-II Solution (Integral curves)
(A)	$y - x\dfrac{dy}{dx} = y^2 + \dfrac{dy}{dx}$	**(P)**	$y = A_1x^2 + A_2x + A_3$
(B)	$(2x - 10y^3)\dfrac{dy}{dx} + y = 0$	**(Q)**	$x^2y^2 + 1 = cy$
(C)	$\left(\dfrac{dy}{dx}\right)\left(\dfrac{d^3y}{dx^3}\right) - 3\left(\dfrac{d^2y}{dx^2}\right)^2 = 0$	**(R)**	$(x+1)(1-y) = cy$
(D)	$(x^2y^2 - 1)\,dy + 2xy^3dx = 0$	**(S)**	$x = A_1y^2 + A_2y + A_3$
		(T)	$xy^2 = 2y^5 + c$

2.

	Column-I		Column-II
(A)	Solution of differential equation $[3x^2y + 2xy - e^x(1+x)]dx + (x^3 + x^2)dy = 0$ is :	**(P)**	$y^2(x^2 + 1 + ce^{x^2}) = 1$
(B)	Solution of differential equation $ydx - xdy - 3xy^2e^{x^2}dx = 0$ is :	**(Q)**	$(x^2 + x^3)y - xe^x = c$
(C)	Solution of differential equation $\dfrac{dy}{dx} = xy(x^2y^2 - 1)$ is :	**(R)**	$\dfrac{x}{y} - \dfrac{3}{2}e^{x^2} = c$
(D)	Solution of differential equation $\dfrac{dy}{dx}(x^2y^3 + xy) = 1$ is :	**(S)**	$\dfrac{1}{x} = 2 - y^2 + ce^{-y^2/2}$
		(T)	$\dfrac{2}{x} = 1 - y^2 + ce^{-y/2}$
	(where c is arbitrary constant).		

Answers

1. A → R ; B → T ; C → S ; D → Q

2. A → Q ; B → R ; C → P ; D → S

Exercise-5 : Subjective Type Problems

1. Find the value of $|a|$ for which the area of triangle included between the coordinate axes and any tangent to the curve $x^a y = \lambda^a$ is constant (where λ is constant.).

2. Let $y = f(x)$ satisfies the differential equation $xy(1+y)\,dx = dy$. If $f(0) = 1$ and $f(2) = \dfrac{e^2}{k - e^2}$, then find the value of k.

3. If $y^2 = 3\cos^2 x + 2\sin^2 x$, then the value of $y^4 + y^3 \dfrac{d^2y}{dx^2}$ is

4. Let $f(x)$ be a differentiable function in $[-1, \infty)$ and $f(0) = 1$ such that
$\lim\limits_{t \to x+1} \dfrac{t^2 f(x+1) - (x+1)^2 f(t)}{f(t) - f(x+1)} = 1$. Find the value of $\lim\limits_{x \to 1} \dfrac{\ln(f(x)) - \ln 2}{x - 1}$.

5. Let $y = (a\sin x + (b+c)\cos x)e^{x+d}$, where a, b, c and d are parameters represent a family of curves, then differential equation for the given family of curves is given by $y'' - \alpha y' + \beta y = 0$, then $\alpha + \beta =$

6. Let $y = f(x)$ satisfies the differential equation $xy\,(1+y)\,dx = dy$. If $f(0) = 1$ and $f(2) = \dfrac{e^2}{k - e^2}$, then find the value of k.

Answers

1.	1	2.	2	3.	6	4.	1	5.	4	6.	2		

❑❑❑

Algebra

8. Quadratic Equations
9. Sequence and Series
10. Determinants
11. Complex Numbers
12. Matrices
13. Permutation and Combinations
14. Binomial Theorem
15. Probability
16. Logarithms

Chapter 8 – Quadratic Equations

8 QUADRATIC EQUATIONS

Exercise-1 : Single Choice Problems

1. Sum of values of x and y satisfying the equation $3^x - 4^y = 77$; $3^{x/2} - 2^y = 7$ is :

   (a) 2 (b) 3 (c) 4 (d) 5

2. If $f(x) = \prod_{i=1}^{3}(x - a_i) + \sum_{i=1}^{3} a_i - 3x$ where $a_i < a_{i+1}$ for $i = 1, 2$, then $f(x) = 0$ has :

   (a) only one distinct real root (b) exactly two distinct real roots
   (c) exactly 3 distinct real roots (d) 3 equal real roots

3. Complete set of real values of 'a' for which the equation $x^4 - 2ax^2 + x + a^2 - a = 0$ has all its roots real :

   (a) $\left[\frac{3}{4}, \infty\right)$ (b) $[1, \infty)$ (c) $[2, \infty)$ (d) $[0, \infty)$

4. The cubic polynomial with leading coefficient unity all whose roots are 3 units less than the roots of the equation $x^3 - 3x^2 - 4x + 12 = 0$ is denoted as $f(x)$, then $f'(x)$ is equal to:

   (a) $3x^2 - 12x + 5$ (b) $3x^2 + 12x + 5$ (c) $3x^2 + 12x - 5$ (d) $3x^2 - 12x - 5$

5. The set of values of $k\,(k \in R)$ for which the equation $x^2 - 4|x| + 3 - |k - 1| = 0$ will have exactly four real roots, is :

   (a) $(-2, 4)$ (b) $(-4, 4)$ (c) $(-4, 2)$ (d) $(-1, 0)$

6. The number of integers satisfying the inequality $\frac{x}{x+6} \le \frac{1}{x}$ is :

   (a) 7 (b) 8 (c) 9 (d) 3

7. The product of uncommon real roots of the two polynomials $p(x) = x^4 + 2x^3 - 8x^2 - 6x + 15$ and $q(x) = x^3 + 4x^2 - x - 10$ is:

   (a) 4 (b) 6 (c) 8 (d) 12

8. If $\lambda_1, \lambda_2\ (\lambda_1 > \lambda_2)$ are two values of λ for which the expression $f(x, y) = x^2 + \lambda xy + y^2 - 5x - 7y + 6$ can be resolved as a product of two linear factors, then the value of $3\lambda_1 + 2\lambda_2$ is :

   (a) 5 (b) 10 (c) 15 (d) 20

9. Let α, β be the roots of the quadratic equation $ax^2 + bx + c = 0$, then the roots of the equation $a(x+1)^2 + b(x+1)(x-2) + c(x-2)^2 = 0$ are :

(a) $\frac{2\alpha+1}{\alpha-1}, \frac{2\beta+1}{\beta-1}$ (b) $\frac{2\alpha-1}{\alpha+1}, \frac{2\beta-1}{\beta+1}$

(c) $\frac{\alpha+1}{\alpha-2}, \frac{\beta+1}{\beta-2}$ (d) $\frac{2\alpha+3}{\alpha-1}, \frac{2\beta+3}{\beta-1}$

10. If $a, b \in R$ distinct numbers satisfying $|a-1|+|b-1|=|a|+|b|=|a+1|+|b+1|$, then the minimum value of $|a-b|$ is :

(a) 3 (b) 0 (c) 1 (d) 2

11. The smallest positive integer p for which expression $x^2 - 2px + 3p + 4$ is negative for atleast one real x is :

(a) 3 (b) 4 (c) 5 (d) 6

12. For $x \in R$, the expression $\frac{x^2 + 2x + c}{x^2 + 4x + 3c}$ can take all real values if $c \in$:

(a) $(1, 2)$ (b) $[0, 1]$

(c) $(0, 1)$ (d) $(-1, 0)$

13. If 2 lies between the roots of the equation $t^2 - mt + 2 = 0$, $(m \in R)$ then the value of $\left[\left(\frac{3|x|}{9+x^2}\right)^m\right]$ is :

(where $[\cdot]$ denotes greatest integer function)

(a) 0 (b) 1 (c) 8 (d) 27

14. The number of integral roots of the equation $x^8 - 24x^7 - 18x^5 + 39x^2 + 1155 = 0$ is :

(a) 0 (b) 2 (c) 4 (d) 6

15. If the value of $m^4 + \frac{1}{m^4} = 119$, then the value of $\left|m^3 - \frac{1}{m^3}\right| =$

(a) 11 (b) 18 (c) 24 (d) 36

16. If the equation $ax^2 + 2bx + c = 0$ and $ax^2 + 2cx + b = 0$, $a \neq 0$, $b \neq c$, have a common root, then their other roots are the roots of the quadratic equation:

(a) $a^2x(x+1) + 4bc = 0$ (b) $a^2x(x+1) + 8bc = 0$

(c) $a^2x(x+2) + 8bc = 0$ (d) $a^2x(1+2x) + 8bc = 0$

17. If $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are the roots of the equation $9x^3 - 9x^2 - x + 1 = 0$; $\alpha, \beta, \gamma \in [0, \pi]$ then the radius of the circle whose centre is $(\Sigma\alpha, \Sigma\cos\alpha)$ and passing through $(2\sin^{-1}(\tan\pi/4), 4)$ is:

(a) 2 (b) 3 (c) 4 (d) 5

18. For real values of x, the value of expression $\frac{11x^2 - 12x - 6}{x^2 + 4x + 2}$:

(a) lies between −17 and −3
(b) does not lie between −17 and −3
(c) lies between 3 and 17
(d) does not lie between 3 and 17

19. $\dfrac{x+3}{x^2-x-2} \geq \dfrac{1}{x-4}$ holds for all x satisfying:

(a) $-2 < x < 1$ or $x > 4$
(b) $-1 < x < 2$ or $x > 4$
(c) $x < -1$ or $2 < x < 4$
(d) $x > -1$ or $2 < x < 4$

20. If $x = 4 + 3i$ (where $i = \sqrt{-1}$), then the value of $x^3 - 4x^2 - 7x + 12$ equals:

(a) -88 (b) $48 + 36i$ (c) $-256 + 12i$ (d) -84

21. Let $f(x) = \dfrac{x^2+x-1}{x^2-x+1}$, then the largest value of $f(x)\ \forall x \in [-1, 3]$ is:

(a) $\dfrac{3}{5}$ (b) $\dfrac{5}{3}$ (c) 1 (d) $\dfrac{4}{3}$

22. In above problem, the range of $f(x)\forall x \in [-1, 1]$ is:

(a) $\left[-1, \dfrac{3}{5}\right]$ (b) $\left[-1, \dfrac{5}{3}\right]$ (c) $\left[-\dfrac{1}{3}, 1\right]$ (d) $[-1, 1]$

23. If the roots of the equation $\dfrac{1}{x+p} + \dfrac{1}{x+q} = \dfrac{1}{r}$ are equal in magnitude but opposite in sign, then the product of the roots is:

(a) $-2(p^2+q^2)$ (b) $-(p^2+q^2)$ (c) $-\dfrac{(p^2+q^2)}{2}$ (d) $-pq$

24. If a root of the equation $a_1x^2 + b_1x + c_1 = 0$ is the reciprocal of a root of the equation $a_2x^2 + b_2x + c_2 = 0$, then :

(a) $(a_1a_2 - c_1c_2)^2 = (a_1b_2 - b_1c_2)(a_2b_1 - b_2c_1)$
(b) $(a_1a_2 - b_1b_2)^2 = (a_1b_2 - b_1c_2)(a_2b_1 - b_2c_1)$
(c) $(b_1c_2 - b_2c_1)^2 = (a_1b_2 - b_1c_2)(a_2b_1 + b_2c_1)$
(d) $(b_1c_2 - b_2c_1)^2 = (a_1b_2 + b_1c_2)(a_2b_1 - b_2c_1)$

25. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$, then the equation with roots $\dfrac{\alpha}{\beta}, \dfrac{\beta}{\alpha}$ is:

(a) $3x^2 - 25x + 3 = 0$
(b) $x^2 + 5x - 3 = 0$
(c) $x^2 - 5x + 3 = 0$
(d) $3x^2 - 19x + 3 = 0$

26. If the difference between the roots of $x^2 + ax + b = 0$ is same as that of $x^2 + bx + a = 0$, $a \neq b$, then:

(a) $a + b + 4 = 0$ (b) $a + b - 4 = 0$ (c) $a - b - 4 = 0$ (d) $a - b + 4 = 0$

27. If $\tan\theta_i$; $i = 1, 2, 3, 4$ are the roots of equation $x^4 - x^3 \sin 2\beta + x^2 \cos 2\beta - x\cos\beta - \sin\beta = 0$, then $\tan(\theta_1 + \theta_2 + \theta_3 + \theta_4) =$

(a) $\sin\beta$ (b) $\cos\beta$ (c) $\tan\beta$ (d) $\cot\beta$

28. Let a, b, c, d are positive real numbers such that $\frac{a}{b} \neq \frac{c}{d}$, then the roots of the equation: $(a^2+b^2)x^2+2x(ac+bd)+(c^2+d^2)=0$ are:

(a) real and distinct
(b) real and equal
(c) imaginary
(d) nothing can be said

29. If α, β are the roots of $ax^2+bx+c=0$, then the equation whose roots are $2+\alpha$, $2+\beta$, is :

(a) $ax^2+x(4a-b)+4a-2b+c=0$
(b) $ax^2+x(4a-b)+4a+2b+c=0$
(c) $ax^2+x(b-4a)+4a+2b+c=0$
(d) $ax^2+x(b-4a)+4a-2b+c=0$

30. Minimum possible number of positive root of the quadratic equation $x^2-(1+\lambda)x+\lambda-2=0$, $\lambda \in R$:

(a) 2
(b) 0
(c) 1
(d) can not be determined

31. Let α, β be real roots of the quadratic equation $x^2+kx+(k^2+2k-4)=0$, then the minimum value of $\alpha^2+\beta^2$ is equal to :

(a) 12
(b) $\frac{4}{9}$
(c) $\frac{16}{9}$
(d) $\frac{8}{9}$

32. Polynomial $P(x)=x^2-ax+5$ and $Q(x)=2x^3+5x-(a-3)$ when divided by $x-2$ have same remainders, then 'a' is equal to:

(a) 10
(b) -10
(c) 20
(d) -20

33. If a and b are non-zero distinct roots of $x^2+ax+b=0$, then the least value of x^2+ax+b is equal to :

(a) $\frac{2}{3}$
(b) $\frac{9}{4}$
(c) $-\frac{9}{4}$
(d) 1

34. Let α, β be the roots of the equation $ax^2+bx+c=0$. A root of the equation $a^3x^2+abcx+c^3=0$ is :

(a) $\alpha+\beta$
(b) $\alpha^2+\beta$
(c) $\alpha^2-\beta$
(d) $\alpha^2\beta$

35. Let a, b, c be the lengths of the sides of a triangle (no two of them are equal) and $k \in R$. If the roots of the equation $x^2+2(a+b+c)x+6k(ab+bc+ca)=0$ are real, then:

(a) $k<\frac{2}{3}$
(b) $k>\frac{2}{3}$
(c) $k>1$
(d) $k<\frac{1}{4}$

36. Root(s) of the equation $9x^2-18|x|+5=0$ belonging to the domain of definition of the function $f(x)=\log(x^2-x-2)$ is/are:

(a) $\frac{-5}{3}, \frac{-1}{3}$
(b) $\frac{5}{3}, \frac{1}{3}$
(c) $\frac{-5}{3}$
(d) $\frac{-1}{3}$

37. If $\beta+\cos^2\alpha, \beta+\sin^2\alpha$ are the roots of $x^2+2bx+c=0$ and $\gamma+\cos^4\alpha$, $\gamma+\sin^4\alpha$ are the roots of $x^2+2Bx+C=0$, then:

(a) $b-B=c-C$
(b) $b^2-B^2=c-C$
(c) $b^2-B^2=4(c-C)$
(d) $4(b^2-B^2)=c-C$

38. Minimum value of $|x-p|+|x-15|+|x-p-15|$. If $p \le x \le 15$ and $0 < p < 15$:
(a) 30 (b) 15 (c) 10 (d) 0

39. If the quadratic equation $4x^2 - 2x - m = 0$ and $4p(q-r)x^2 - 2q(r-p)x + r(p-q) = 0$ have a common root such that second equation has equal roots then the value of m will be :
(a) 0 (b) 1 (c) 2 (d) 3

40. The range of k for which the inequality $k\cos^2 x - k\cos x + 1 \ge 0 \forall x \in (-\infty, \infty)$ is :
(a) $k > -\frac{1}{2}$ (b) $k > 4$ (c) $-\frac{1}{2} \le k \le 4$ (d) $\frac{1}{2} \le k \le 5$

41. If $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$ are roots of the cubic equation $f(x) = 0$ where α, β, γ are the roots of the cubic equation $3x^3 - 2x + 5 = 0$, then the number of negative real roots of the equation $f(x) = 0$ is:
(a) 0 (b) 1 (c) 2 (d) 3

42. The sum of all integral values of λ for which $(\lambda^2 + \lambda - 2)x^2 + (\lambda + 2)x < 1 \forall x \in R$, is :
(a) −1 (b) −3 (c) 0 (d) −2

43. If $\alpha, \beta, \gamma, \delta \in R$ satisfy $\frac{(\alpha+1)^2 + (\beta+1)^2 + (\gamma+1)^2 + (\delta+1)^2}{\alpha+\beta+\gamma+\delta} = 4$
If biquadratic equation $a_0x^4 + a_1x^3 + a_2x^2 + a_3x + a_4 = 0$ has the roots $\left(\alpha + \frac{1}{\beta} - 1\right), \left(\beta + \frac{1}{\gamma} - 1\right), \left(\gamma + \frac{1}{\delta} - 1\right), \left(\delta + \frac{1}{\alpha} - 1\right)$. Then the value of a_2/a_0 is :
(a) 4 (b) −4 (c) 6 (d) none of these

44. If the complete set of value of x satisfying $|x-1|+|x-2|+|x-3| \ge 6$ is $(-\infty, a] \cup [b, \infty)$, then $a + b =$:
(a) 2 (b) 3 (c) 6 (d) 4

45. If exactly one root of the quadratic equation $x^2 - (a+1)x + 2a = 0$ lies in the interval (0, 3), then the set of value 'a' is given by:
(a) $(-\infty, 0) \cup (6, \infty)$ (b) $(-\infty, 0] \cup (6, \infty)$
(c) $(-\infty, 0] \cup [6, \infty)$ (d) $(0, 6)$

46. The condition that the root of $x^3 + 3px^2 + 3qx + r = 0$ are in H.P. is :
(a) $2p^3 - 3pqr + r^2 = 0$ (b) $3p^3 - 2pqr + p^2 = 0$
(c) $2q^3 - 3pqr + r^2 = 0$ (d) $r^3 - 3pqr + 2q^3 = 0$

47. If x is real and $4y^2 + 4xy + x + 6 = 0$, then the complete set of values of x for which y is real, is:
(a) $x \le -2$ or $x \ge 3$ (b) $x \le 2$ or $x \ge 3$ (c) $x \le -3$ or $x \ge 2$ (d) $-3 \le x \le 2$

48. The solution of the equation $\log_{\cos x^2}(3-2x) < \log_{\cos x^2}(2x-1)$ is :
(a) (1/2, 1) (b) $(-\infty, 1)$
(c) (1/2, 3) (d) $(1, \infty) - \sqrt{2n\pi}, n \in N$

49. If the roots α, β of the equation $px^2 + qx + r = 0$ are real and of opposite sign (where p, q, r are real coefficient), then the roots of the equation $\alpha(x-\beta)^2 + \beta(x-\alpha)^2 = 0$ are:

(a) positive (b) negative
(c) real and of opposite sign (d) imaginary

50. Let a, b and c be three distinct real roots of the cubic $x^3 + 2x^2 - 4x - 4 = 0$.
If the equation $x^3 + qx^2 + rx + s = 0$ has roots $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$, then the value of $(q + r + s)$ is equal to :

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

51. Solution set of the inequality, $2 - \log_2(x^2 + 3x) \geq 0$ is :

(a) $[-4, 1]$ (b) $[-4, -3) \cup (0, 1]$ (c) $(-\infty, -3) \cup (1, \infty)$ (d) $(-\infty, -4) \cup [1, \infty)$

52. For what least integral 'k' is the quadratic trinomial $(k-2)x^2 + 8x + (k+4)$ is positive for all real values of x ?

(a) $k = 4$ (b) $k = 5$ (c) $k = 3$ (d) $k = 6$

53. If roots of the equation $(m-2)x^2 - (8-2m)x - (8-3m) = 0$ are opposite in sign, then number of integral values(s) of m is/are :

(a) 0 (b) 1 (c) 2 (d) more than 2

54. If $\log_{0.6}\left(\log_6\left(\frac{x^2+x}{x+4}\right)\right) < 0$, then complete set of value of 'x' is :

(a) $(-4, -3) \cup (8, \infty)$ (b) $(-\infty, -3) \cup (8, \infty)$
(c) $(8, \infty)$ (d) None of these

55. Two different real numbers α and β are the roots of the quadratic equation $ax^2 + c = 0$ with $a, c \neq 0$, then $\alpha^3 + \beta^3$ is :

(a) a (b) $-c$ (c) 0 (d) -1

56. The least integral value of 'k' for which $(k-1)x^2 - (k+1)x + (k+1)$ is positive for all real value of x is:

(a) 1 (b) 2 (c) 3 (d) 4

57. If $(-2, 7)$ is the highest point on the graph of $y = -2x^2 - 4ax + k$, then k equals:

(a) 31 (b) 11 (c) -1 (d) $-1/3$

58. If $a + b + c = 0$, $a, b, c \in Q$ then roots of the equation $(b+c-a)x^2 + (c+a-b)x + (a+b-c) = 0$ are :

(a) rational (b) irrational (c) imaginary (d) none of these

59. If two roots of $x^3 - ax^2 + bx - c = 0$ are equal in magnitude but opposite in sign. Then:

(a) $a + bc = 0$ (b) $a^2 = bc$
(c) $ab = c$ (d) $a - b + c = 0$

60. If α and β are the real roots of $x^2 + px + q = 0$ and α^4, β^4 are the roots of $x^2 - rx + s = 0$. Then the equation $x^2 - 4qx + 2q^2 - r = 0$ has always ($\alpha \neq \beta, p \neq 0, p, q, r, s \in R$):

(a) one positive and one negative root (b) two positive roots

(c) two negative roots (d) can't say anything

61. If $x^2 + px + 1$ is a factor of $ax^3 + bx + c$, then:

(a) $a^2 + c^2 = -ab$ (b) $a^2 + c^2 = ab$ (c) $a^2 - c^2 = ab$ (d) $a^2 - c^2 = -ab$

62. In a ΔABC $\tan\frac{A}{2}$, $\tan\frac{B}{2}$, $\tan\frac{C}{2}$ are in H.P., then the value of $\cot\frac{A}{2}\cot\frac{C}{2}$ is :

(a) 3 (b) 2 (c) 1 (d) $\sqrt{3}$

63. Let $f(x) = 10 - |x - 10|\ \forall x \in [-9, 9]$, if M and m be the maximum and minimum value of $f(x)$ respectively, then :

(a) $M + m = 0$ (b) $2M + m = -9$ (c) $2M + m = 7$ (d) $M + m = 7$

64. Solution of the quadratic equation $(3|x|-3)^2 = |x| + 7$, which belongs to the domain of the function $y = \sqrt{(x-4)x}$ is :

(a) $\pm\frac{1}{9}, \pm 2$ (b) $\frac{1}{9}, 8$ (c) $-2, -\frac{1}{9}$ (d) $-\frac{1}{9}, 8$

65. Number of real solutions of the equation $x^2 + 3|x| + 2 = 0$ is :

(a) 0 (b) 1 (c) 2 (d) 4

66. If the roots of equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c =$

(a) 3 (b) −2 (c) 1 (d) 2

67. If x is real, then maximum value of $\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$ is :

(a) 41 (b) 1 (c) $\frac{17}{7}$ (d) $\frac{1}{4}$

68. If $\frac{x^2 + 2x + 7}{2x + 3} < 6,\ x \in R$ then :

(a) $x \in \left(-\infty, -\frac{3}{2}\right) \cup (11, \infty)$ (b) $x \in (-\infty, -1) \cup (11, \infty)$

(c) $x \in \left(-\frac{3}{2}, -1\right)$ (d) $x \in \left(-\infty, -\frac{3}{2}\right) \cup (-1, 11)$

69. If x is real, then range of $\frac{3x - 2}{7x + 5}$ is :

(a) $R - \left\{\frac{2}{5}\right\}$ (b) $R - \left\{\frac{3}{7}\right\}$ (c) $(-\infty, \infty)$ (d) $R - \left\{\frac{-2}{5}\right\}$

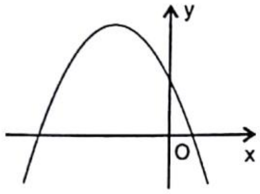
70. Let A denotes the set of values of x for which $\frac{x+2}{x-4} \leq 0$ and B denotes the set of values of x for which $x^2 - ax - 4 \leq 0$. If B is the subset of A, then a **CAN NOT** take the integral value :

(a) 0 (b) 1 (c) 2 (d) 3

71. If the quadratic polynomial $P(x) = (p-3)x^2 - 2px + 3p - 6$ ranges from $[0, \infty)$ for every $x \in R$, then the value of p can be :

(a) 3 (b) 4 (c) 6 (d) 7

72. If graph of the quadratic $y = ax^2 + bx + c$ is given below :



then :

(a) $a < 0,\ b > 0, c > 0$ (b) $a < 0,\ b > 0, c < 0$

(c) $a < 0,\ b < 0, c > 0$ (d) $a < 0,\ b < 0, c < 0$

73. If quadratic equation $ax^2 + bx + c = 0$ does not have real roots, then which of the following may be false :

(a) $a(a-b+c) > 0$ (b) $c(a-b+c) > 0$

(c) $b(a-b+c) > 0$ (d) $(a+b+c)(a-b+c) > 0$

74. Minimum value of $y = x^2 - 3x + 5,\ x \in [-4, 1]$ is :

(a) 3 (b) $\frac{11}{4}$ (c) 0 (d) 9

75. If $3x^2 - 17x + 10 = 0$ and $x^2 - 5x + m = 0$ has a common root, then sum of all possible real values of 'm' is :

(a) 0 (b) $-\frac{26}{9}$ (c) $\frac{29}{9}$ (d) $\frac{26}{3}$

76. For real numbers x and y, if $x^2 + xy - y^2 + 2x - y + 1 = 0$, then :

(a) y can not be between 0 and $\frac{8}{5}$ (b) y can not be between $-\frac{8}{5}$ and $\frac{8}{5}$

(c) y can not be between $-\frac{8}{5}$ and 0 (d) y can not be between $-\frac{16}{5}$ and 0

77. If $3x^4 - 6x^3 + kx^2 - 8x - 12$ is divisible by $x - 3$, then it is also divisible by :

(a) $3x^2 - 4$ (b) $3x^2 + 4$

(c) $3x^2 + x$ (d) $3x^2 - x$

78. The complete set of values of a so that equation $\sin^4 x + a\sin^2 x + 4 = 0$ has at least one real root is :

(a) $(\infty, -5]$ (b) $(-\infty, 4] \cup [4, \infty)$

(c) $(-\infty, -4]$ (d) $[4, \infty)$

79. Let r, s, t be the roots of the equation $x^3 + ax^2 + bx + c = 0$, such that $(rs)^2 + (st)^2 + (rt)^2 = b^2 - kac$, then $k =$

(a) 1 (b) 2 (c) 3 (d) 4

80. If the roots of the cubic $x^3 + ax^2 + bx + c = 0$ are three consecutive positive integers, then the value of $\frac{a^2}{b+1} =$

(a) 1 (b) 2 (c) 3 (d) 4

81. Let 'k' be a real number. The minimum number of distinct real roots possible of the equation $(3x^2 + kx + 3)(x^2 + kx - 1) = 0$ is :

(a) 0 (b) 2 (c) 3 (d) 4

82. If r and s are variables satisfying the equation $\frac{1}{r+s} = \frac{1}{r} + \frac{1}{s}$. The value of $\left(\frac{r}{s}\right)^3$ is equal to :

(a) 1 (b) −1
(c) 3 (d) not possible to determine

83. Let $f(x) = x^2 + ax + b$. If the maximum and the minimum values of $f(x)$ are 3 and 2 respectively for $0 \le x \le 2$, then the possible ordered pair of (a, b) is :

(a) (−2, 3) (b) (−3/2 , 2) (c) (−5/2, 3) (d) (−5/2 , 2)

84. The roots of the equation $|x^2 - x - 6| = x + 2$ are given by :

(a) −2, 2, 4 (b) 0, 1, 4 (c) −2, 1, 4 (d) 0, 2, 4

85. If a, b, c be the sides of ΔABC and equations $ax^2 + bx + c = 0$ and $5x^2 + 12x + 13 = 0$ have a common root, then $\angle C$ is :

(a) 60° (b) 90° (c) 120° (d) 45°

86. If α, β and γ are three real roots of the equation $x^3 - 6x^2 + 5x - 1 = 0$, then the value of $\alpha^4 + \beta^4 + \gamma^4$ is :

(a) 250 (b) 650 (c) 150 (d) 950

87. If one of the roots of the equation $2x^2 - 6x + k = 0$ is $\frac{\alpha + 5i}{2}$, then the value of α and k are :

(a) $\alpha = 3, k = 8$ (b) $\alpha = \frac{3}{2}, k = 17$ (c) $\alpha = -3, k = -17$ (d) $\alpha = 3, k = 17$

88. Let x_1 and x_2 be the real roots of the equation $x^2 - (k-2)x + (k^2 + 3k + 5) = 0$, then the maximum value of $x_1^2 + x_2^2$ is :

(a) 19 (b) 18 (c) $\frac{50}{9}$ (d) non-existent

89. The complete set of values of 'a' for which the inequality $(a-1)\,x^2 - (a+1)\,x + (a-1) \ge 0$ is true for all $x \ge 2$.

(a) $\left(\frac{3}{7}, 1\right]$ (b) $(-\infty, 1)$ (c) $\left(-\infty, \frac{7}{3}\right]$ (d) $\left[\frac{7}{3}, \infty\right)$

90. If α, β be the roots of $4x^2 - 17x + \lambda = 0$, $\lambda \in R$ such that $1 < \alpha < 2$ and $2 < \beta < 3$, then the number of integral values of λ is :

(a) 1 (b) 2 (c) 3 (d) 4

91. Assume that p is a real number. In order of $\sqrt[3]{x+3p+1}-\sqrt[3]{x}=1$ to have real solutions, it is necessary that:

(a) $p \geq 1/4$ (b) $p \geq -1/4$ (c) $p \geq 1/3$ (d) $p \geq -1/3$

92. If α, β are the roots of the quadratic equation $x^2-\left(3+2^{\sqrt{\log_2 3}}-3^{\sqrt{\log_3 2}}\right)x-2(3^{\log_3 2}-2^{\log_2 3})=0$, then the value of $\alpha^2+\alpha\beta+\beta^2$ is equal to :

(a) 3 (b) 5 (c) 7 (d) 11

93. The minimum value of $f(x, y)=x^2-4x+y^2+6y$ when x and y are subjected to the restrictions $0 \leq x \leq 1$ and $0 \leq y \leq 1$, is :

(a) -1 (b) -2 (c) -3 (d) -5

94. The expression $ax^2+2bx+c$, where 'a' is non-zero real number, has same sign as that of 'a' for every real value of x, then roots of quadratic equation $ax^2+(b-c)\,x-2b-c-a=0$, are :

(a) real and equal
(b) real and unequal
(c) non-real having positive real part
(d) non-real having negative real part

95. Let a, b and c be the roots of $x^3-x+1=0$, then the value of $\left(\frac{1}{a+1}+\frac{1}{b+1}+\frac{1}{c+1}\right)$ equals to :

(a) 1 (b) -1 (c) 2 (d) -2

96. The number of integral values of k for which the inequality $x^2-2(4k-1)x+15k^2-2k-7 \geq 0$ holds for all $x \in R$ is :

(a) 2 (b) 3 (c) 4 (d) infinite

97. The number of integral values which can be taken by the expression, $f(x)=\dfrac{x^3-1}{(x-1)\,(x^2-x+1)}$ for $x \in R$, is :

(a) 1 (b) 2 (c) 3 (d) infinite

98. The complete set of values of m for which the inequality $\dfrac{x^2-mx-2}{x^2+mx+4}>-1$ is satisfied $\forall x \in R$, is :

(a) $m=0$ (b) $-1<m<1$ (c) $-2<m<2$ (d) $-4<m<4$

99. The complete set of values of a for which the roots of the equation $x^2-2|a+1|x+1=0$ are real is given by :

(a) $(-\infty, -2] \cup [0, \infty)$
(b) $(-\infty, -1] \cup [0, \infty)$
(c) $(-\infty, -1] \cup [1, \infty)$
(d) $(-\infty, -2] \cup [1, \infty)$

100. The quadratic polynomials defined on real coefficients $P(x)=a_1x^2+2b_1x+c_1$, $Q(x)=a_2x^2+2b_2x+c_2 \cdot P(x)$ and $Q(x)$ both take positive values $\forall x \in R$. If $f(x)=a_1a_2x^2+b_1b_2x+c_1c_2$, then :

(a) $f(x)<0\,\forall\, x \in R$
(b) $f(x)>0\,\forall\, x \in R$

(c) $f(x)$ takes both positive and negative values

(d) Nothing can be said about $f(x)$

101. If the equation $x^2 + 4 + 3\cos(ax+b) = 2x$ has a solution then a possible value of $(a+b)$ equals

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) π

102. Let α, β be the roots of $x^2 - 4x + A = 0$ and γ, δ be the roots of $x^2 - 36x + B = 0$. If $\alpha, \beta, \gamma, \delta$ form an increasing G.P. and $A^t = B$ then the value of 't' equals

(a) 4 (b) 5 (c) 6 (d) 8

103. How many roots does the following equation possess $3^{|x|}\,(|\,2-|x|\,|) = 1$?

(a) 2 (b) 3 (c) 4 (d) 6

104. If $\cot\alpha$ equals the integral solution of inequality $4x^2 - 16x + 15 < 0$ and $\sin\beta$ equals to the slope of the bisector of the first quadrant, then $\sin(\alpha+\beta)\sin(\alpha-\beta)$ is equal to :

(a) $-\frac{3}{5}$ (b) $-\frac{4}{5}$ (c) $\frac{2}{\sqrt{2}}$ (d) 3

105. Consider the functions $f_1(x) = x$ and $f_2(x) = 2 + \log_e x,\ x > 0$, where e is the base of natural logarithm. The graphs of the functions intersect :

(a) once in (0, 1) and never in $(1, \infty)$ (b) once in (0, 1) and once in (e^2, ∞)

(c) once in (0, 1) and once in (e, e^2) (d) more than twice in $(0, \infty)$

106. The sum of all the real roots of equation

$$x^4 - 3x^3 - 2x^2 - 3x + 1 = 0 \text{ is :}$$

(a) 1 (b) 2 (c) 3 (d) 4

107. If $\alpha, \beta\ (\alpha < \beta)$ are the real roots of the equation $x^2 - (k+4)x + k^2 - 12 = 0$ such that $4 \in (\alpha, \beta)$; then the number of integral values of k equal to :

(a) 4 (b) 5 (c) 6 (d) 7

108. Let α, β be real roots of the quadratic equation $x^2 + kx + (k^2 + 2k - 4) = 0$, then the maximum value of $(\alpha^2 + \beta^2)$ is equal to :

(a) 9 (b) 10 (c) 11 (d) 12

109. Let $f(x) = a^x - x\ln a,\ a > 1$. Then the complete set of real values of x for which $f'(x) > 0$ is :

(a) $(1, \infty)$ (b) $(-1, \infty)$ (c) $(0, \infty)$ (d) $(0, 1)$

110. If a, b and c are the roots of the equation $x^3 + 2x^2 + 1 = 0$, find $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$:

(a) 8 (b) –8 (c) 0 (d) 2

111. Let α, β are the two real roots of equation $x^2 + px + q = 0,\ p, q \in R,\ q \neq 0$. If the quadratic equation $g(x) = 0$ has two roots $\alpha + \frac{1}{\alpha}, \beta + \frac{1}{\beta}$ such that sum of roots is equal to product of roots, then the complete range of q is :

(a) $\left[\frac{1}{3}, 3\right]$ (b) $\left(\frac{1}{3}, 3\right]$ (c) $\left[\frac{1}{3}, 3\right)$ (d) $\left(-\infty, \frac{1}{3}\right) \cup (3, \infty)$

112. If the equation $\ln(x^2 + 5x) - \ln(x + a + 3) = 0$ has exactly one solution for x, then number of integers in the range of a is :

(a) 4 (b) 5 (c) 6 (d) 7

113. Let $f(x) = x^2 + \frac{1}{x^2} - 6x - \frac{6}{x} + 2$, then minimum value of $f(x)$ is :

(a) -2 (b) -8 (c) -9 (d) -12

114. If $x^2 + bx + b$ is a factor of $x^3 + 2x^2 + 2x + c\,(c \neq 0)$, then $b - c$ is :

(a) 2 (b) -1 (c) 0 (d) -2

115. If roots of $x^3 + 2x^2 + 1 = 0$ are α, β and γ, then the value of $(\alpha\beta)^3 + (\beta\gamma)^3 + (\alpha\gamma)^3$, is :

(a) -11 (b) 3 (c) 0 (d) -2

116. How many roots does the following equation possess $3^{|x|}(|2 - |x||) = 1$?

(a) 2 (b) 3 (c) 4 (d) 6

117. The sum of all the real roots of equation $x^4 - 3x^3 - 2x^2 - 3x + 1 = 0$ is :

(a) 1 (b) 2 (c) 3 (d) 4

118. If α and β are the roots of the quadratic equation $4x^2 + 2x - 1 = 0$ then the value of $\sum_{r=1}^{\infty}(\alpha^r + \beta^r)$ is :

(a) 2 (b) 3 (c) 6 (d) 0

119. The number of value(s) of x satisfying the equation $(2011)^x + (2012)^x + (2013)^x - (2014)^x = 0$ is/are :

(a) exactly 2 (b) exactly 1 (c) more than one (d) 0

120. If $\alpha, \beta\,(\alpha < \beta)$ are the real roots of the equation $x^2 - (k+4)x + k^2 - 12 = 0$ such that $4 \in (\alpha, \beta)$; then the number of integral values of k equals to :

(a) 4 (b) 5 (c) 6 (d) 7

121. Let α, β be real roots of the quadratic equation $x^2 + kx + (k^2 + 2k - 4) = 0$, then the maximum value of $(\alpha^2 + \beta^2)$ is equal to :

(a) 9 (b) 10 (c) 11 (d) 12

122. The exhaustive set of values of a for which inequation $(a-1)x^2 - (a+1)x + a - 1 \geq 0$ is true $\forall\, x \geq 2$.

(a) $(-\infty, 1)$ (b) $\left[\frac{7}{3}, \infty\right)$ (c) $\left[\frac{3}{7}, \infty\right)$ (d) None of these

123. If the equation $x^2 + ax + 12 = 0$, $x^2 + bx + 15 = 0$ and $x^2 + (a+b)x + 36 = 0$ have a common positive root, then $b - 2a$ is equal to.

(a) -6 (b) 22 (c) 6 (d) -22

124. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has

(a) infinite number of real roots (b) no real root

(c) exactly one real root (d) exactly four real roots

125. The difference between the maximum and minimum value of the function $f(x) = 3\sin^4 x - \cos^6 x$ is :

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) 3 (d) 4

126. If α, β are the roots of $x^2 - 3x + \lambda = 0$ $(\lambda \in R)$ and $\alpha < 1 < \beta$, then the true set of values of λ equals :

(a) $\lambda \in \left(2, \frac{9}{4}\right]$ (b) $\lambda \in \left(-\infty, \frac{9}{4}\right]$ (c) $\lambda \in (2, \infty)$ (d) $\lambda \in (-\infty, 2)$

127. If $2x^2 + 5x + 7 = 0$ and $ax^2 + bx + c = 0$ have at least one root common such that $a, b, c \in \{1, 2, \ldots\ldots, 100\}$, then the difference between the maximum and minimum values of $a + b + c$ is :

(a) 196 (b) 284 (c) 182 (d) 126

128. Two particles, A and B, are in motion in the xy-plane. Their co-ordinates at each instant of time $t (t \geq 0)$ are given by $x_A = t$, $y_A = 2t$, $x_B = 1 - t$ and $y_B = t$. The minimum distance between particles A and B is :

(a) $\frac{1}{5}$ (b) $\frac{1}{\sqrt{5}}$ (c) 1 (d) $\sqrt{\frac{2}{3}}$

129. If $a \neq 0$ and the equation $ax^2 + bx + c = 0$ has two roots α and β such that $\alpha < -3$ and $\beta > 2$, which of the following is always true ?

(a) $a(a + |b| + c) > 0$ (b) $a(a + |b| + c) < 0$

(c) $9a - 3b + c > 0$ (d) $(9a - 3b + c)(4a + 2b + c) < 0$

130. The number of negative real roots of the equation $(x^2 + 5x)^2 - 24 = 2(x^2 + 5x)$ is :

(a) 4 (b) 3 (c) 2 (d) 1

131. The number of real values of x satisfying the equation $3|x - 2| + |1 - 5x| + 4|3x + 1| = 13$ is :

(a) 1 (b) 4 (c) 2 (d) 3

132. If $\log_{\cos x} \sin x \geq 2$ and $0 \leq x \leq 3\pi$ then $\sin x$ lies in the interval

(a) $\left[\frac{\sqrt{5}-1}{2}, 1\right]$ (b) $\left[0, \frac{\sqrt{5}-1}{2}\right]$ (c) $\left[\frac{1}{2}, 1\right]$ (d) none of these

133. Let $f(x) = x^2 + bx + c$, minimum value of $f(x)$ is -5, then absolute value of the difference of the roots of $f(x)$ is :

(a) 5 (b) $\sqrt{20}$

(c) $\sqrt{15}$ (d) Can't be determined

134. Sum of all the solutions of the equation $|x - 3| + |x + 5| = 7x$, is :

(a) $\frac{6}{7}$ (b) $\frac{8}{7}$ (c) $\frac{58}{63}$ (d) $\frac{8}{45}$

135. Let $f(x) = x^2 + \frac{1}{x^2} - 6x - \frac{6}{x} + 2$, then minimum value of $f(x)$ is :

(a) -2 (b) -8 (c) -9 (d) -12

136. If $a + b + c = 1, a^2 + b^2 + c^2 = 9$ and $a^3 + b^3 + c^3 = 1$, then the value of $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ is :

(a) $\frac{2}{3}$ (b) 5 (c) 6 (d) 1

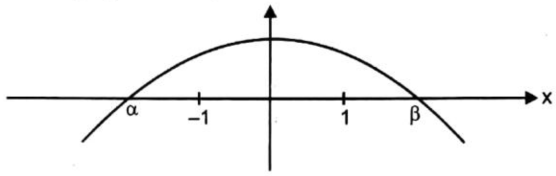
137. If roots of $x^3 + 2x^2 + 1 = 0$ are α, β and γ, then the value of $(\alpha\beta)^3 + (\beta\gamma)^3 + (\alpha\gamma)^3$, is :

(a) -11 (b) 3 (c) 0 (d) -2

138. If $x^2 + bx + b$ is a factor of $x^3 + 2x^2 + 2x + c$ $(c \neq 0)$, then $b - c$ is :

(a) 2 (b) -1 (c) 0 (d) -2

139. The graph of quadratic polynomical $f(x) = ax^2 + bx + c$ is shown below



(a) $\frac{c}{a}|\beta - \alpha| < -2$ (b) $f(x) > 0 \ \forall \ x > \beta$ (c) $ac > 0$ (d) $\frac{c}{a} > -1$

140. If $f(x) = \frac{x^2 - 3x + 4}{x^2 + 3x + 4}$, then complete solution of $0 < f(x) < 1$, is :

(a) $(-\infty, \infty)$ (b) $(0, \infty)$ (c) $(-\infty, 0)$ (d) $(0, 1) \cup (2, \infty)$

141. If α, β, γ are the roots of the equation $x^3 + 2x^2 - x + 1 = 0$, then value of $\frac{(2-\alpha)(2-\beta)(2-\gamma)}{(2+\alpha)(2+\beta)(2+\gamma)}$, is:

(a) 5 (b) -5 (c) 10 (d) $\frac{5}{3}$

142. If α and β are roots of the quadratic equation $x^2 + 4x + 3 = 0$, then the equation whose roots are $2\alpha + \beta$ and $\alpha + 2\beta$ is :

(a) $x^2 - 12x + 35 = 0$ (b) $x^2 + 12x - 33 = 0$ (c) $x^2 - 12x - 33 = 0$ (d) $x^2 + 12x + 35 = 0$

143. If a, b, c are real distinct numbers such that $a^3 + b^3 + c^3 = 3abc$, then the quadratic equation $ax^2 + bx + c = 0$ has

(a) Real roots (b) At least one negative root

(c) Both roots are negative (d) Non real roots

144. If the equation $x^2 + ax + 12 = 0$, $x^2 + bx + 15 = 0$ and $x^2 + (a + b)x + 36 = 0$ have a common positive root, then $b - 2a$ is equal to.

(a) -6 (b) 22 (c) 6 (d) -22

145. Consider the equation $x^3 - ax^2 + bx - c = 0$, where a, b, c are rational number, $a \neq 1$. It is given that x_1, x_2 and $x_1 x_2$ are the real roots of the equation. Then $x_1 x_2\left(\frac{a+1}{b+c}\right) =$

(a) 1 (b) 2 (c) 3 (d) 4

146. The exhaustive set of values of a for which inequation $(a-1)x^2 - (a+1)x + a - 1 \geq 0$ is true $\forall\, x \geq 2$.

(a) $(-\infty, 1)$ (b) $\left[\frac{7}{3}, \infty\right)$ (c) $\left[\frac{3}{7}, \infty\right)$ (d) None of these

147. The number of real solutions of the equation

$$x^2 - 3|x| + 2 = 0$$

(a) 2 (b) 4 (c) 1 (d) 3

148. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has

(a) infinite number of real roots (b) no real root
(c) exactly one real root (d) exactly four real roots

149. If α, β are the roots of the quadratic equation $x^2 - 2(1 - \sin 2\theta)x - 2\cos^2 2\theta = 0$, $(\theta \in R)$ then the minimum value of $(\alpha^2 + \beta^2)$ is equal to :

(a) -4 (b) 8 (c) 0 (d) 2

150. If the equation $|\sin x|^2 + |\sin x| + b = 0$ has two distinct roots in $[0, \pi]$; then the number of integers in the range of b is equals to :

(a) 0 (b) 1 (c) 2 (d) 3

151. If $a \neq 0$ and the equation $ax^2 + bx + c = 0$ has two roots α and β such that $\alpha < -3$ and $\beta > 2$. Which of the following is always true ?

(a) $a(a + |b| + c) > 0$ (b) $a(a + |b| + c) < 0$
(c) $9a - 3b + c > 0$ (d) $(9a - 3b + c)(4a + 2b + c) < 0$

152. If α, β are the roots of the quadratic equation $x^2 + px + q = 0$ and γ, δ are the roots of $x^2 + px - r = 0$ then $(\alpha - \gamma)(\alpha - \delta)$ is equal to :

(a) $q + r$ (b) $q - r$ (c) $-(q + r)$ (d) $-(p + q + r)$

153. Complete set of solution of $\log_{1/3}(2^{x+2} - 4^x) \geq -2$ is :

(a) $(-\infty, 2)$ (b) $(-\infty, 2 + \sqrt{13})$ (c) $(2, \infty)$ (d) None of these

Answers

1. (d)	**2.** (c)	**3.** (a)	**4.** (b)	**5.** (a)	**6.** (a)	**7.** (b)	**8.** (c)	**9.** (a)	**10.** (d)
11. (c)	**12.** (c)	**13.** (a)	**14.** (a)	**15.** (d)	**16.** (d)	**17.** (b)	**18.** (b)	**19.** (c)	**20.** (a)
21. (b)	**22.** (d)	**23.** (c)	**24.** (a)	**25.** (d)	**26.** (a)	**27.** (d)	**28.** (c)	**29.** (d)	**30.** (c)
31. (d)	**32.** (d)	**33.** (c)	**34.** (d)	**35.** (a)	**36.** (c)	**37.** (b)	**38.** (b)	**39.** (c)	**40.** (c)
41. (b)	**42.** (b)	**43.** (c)	**44.** (d)	**45.** (b)	**46.** (c)	**47.** (a)	**48.** (a)	**49.** (c)	**50.** (c)
51. (b)	**52.** (b)	**53.** (a)	**54.** (a)	**55.** (c)	**56.** (b)	**57.** (c)	**58.** (a)	**59.** (c)	**60.** (a)
61. (c)	**62.** (a)	**63.** (a)	**64.** (c)	**65.** (a)	**66.** (c)	**67.** (a)	**68.** (d)	**69.** (b)	**70.** (d)
71. (c)	**72.** (c)	**73.** (c)	**74.** (a)	**75.** (c)	**76.** (c)	**77.** (b)	**78.** (a)	**79.** (b)	**80.** (c)
81. (b)	**82.** (a)	**83.** (a)	**84.** (a)	**85.** (b)	**86.** (b)	**87.** (d)	**88.** (b)	**89.** (d)	**90.** (b)
91. (b)	**92.** (c)	**93.** (c)	**94.** (b)	**95.** (d)	**96.** (b)	**97.** (b)	**98.** (d)	**99.** (a)	**100.** (b)
101. (d)	**102.** (b)	**103.** (c)	**104.** (b)	**105.** (c)	**106.** (d)	**107.** (d)	**108.** (d)	**109.** (c)	**110.** (a)
111. (a)	**112.** (b)	**113.** (c)	**114.** (c)	**115.** (b)	**116.** (c)	**117.** (d)	**118.** (d)	**119.** (b)	**120.** (d)
121. (d)	**122.** (b)	**123.** (c)	**124.** (b)	**125.** (d)	**126.** (d)	**127.** (c)	**128.** (b)	**129.** (b)	**130.** (b)
131. (c)	**132.** (b)	**133.** (b)	**134.** (b)	**135.** (c)	**136.** (d)	**137.** (b)	**138.** (c)	**139.** (a)	**140.** (b)
141. (b)	**142.** (d)	**143.** (a)	**144.** (c)	**145.** (a)	**146.** (b)	**147.** (b)	**148.** (b)	**149.** (c)	**150.** (c)
151. (b)	**152.** (c)	**153.** (a)							

Exercise-2 : One or More than One Answer Is/are Correct

1. Let S is the set of all real x such that $\frac{2x-1}{2x^3+3x^2+x}$ is positive, then S contains :
   (a) $\left(-\infty,-\frac{3}{2}\right)$ (b) $\left(-\frac{3}{2},-\frac{1}{2}\right)$ (c) $\left(-\frac{1}{2},0\right)$ (d) $\left(\frac{1}{2},2\right)$

2. If $kx^2-4x+3k+1>0$ for atleast one $x>0$, then if $k\in S$, then S contains:
   (a) $(1,\infty)$ (b) $(0,\infty)$ (c) $(-1,\infty)$ (d) $\left(-\frac{1}{4},\infty\right)$

3. The equation $|x^2-x-6|=x+2$ has:
   (a) two positive roots (b) two real roots
   (c) three real roots (d) four real roots

4. If the roots of the equation $x^2-ax-b=0$ $(a,\ b\in R)$ are both lying between -2 and 2, then :
   (a) $|a|<2-\frac{b}{2}$ (b) $|a|>2-\frac{b}{2}$
   (c) $|a|<4$ (d) $|a|>\frac{b}{2}-2$

5. Consider the equation in real number x and a real parameter λ, $|x-1|-|x-2|+|x-4|=\lambda$ Then for $\lambda\geq 1$, the number of solutions, the equation can have is/are :
   (a) 1 (b) 2 (c) 3 (d) 4

6. If a and b are two distinct non-zero real numbers such that $a-b=\frac{a}{b}=\frac{1}{b}-\frac{1}{a}$, then :
   (a) $a>0$ (b) $a<0$ (c) $b<0$ (d) $b>0$

7. Let $f(x)=ax^2+bx+c$, $a>0$ and $f(2-x)=f(2+x)\forall x\in R$ and $f(x)=0$ has 2 distinct real roots, then which of the following is true ?
   (a) Atleast one root must be positive
   (b) $f(2)<f(0)>f(1)$
   (c) Minimum value of $f(x)$ is negative
   (d) Vertex of graph of $y=f(x)$ lies in 3rd quadrat

8. In the above problem, if roots of equation $f(x)=0$ are non-real complex, then which of the following is false ?
   (a) $f(x)=\sin\frac{\pi x}{4}$ must have 2 solutions
   (b) $4a-2b+c<0$
   (c) If $\log_{f(2)} f(3)$ is not defined, then $f(x)\geq 1\ \forall\ x\in R$
   (d) All a,b,c are positive

9. If exactly two integers lie between the roots of equation $x^2+ax-1=0$. Then integral value(s) of 'a' is/are :
   (a) -1 (b) -2 (c) 1 (d) 2

10. If the minimum value of the quadratic expression $y = ax^2 + bx + c$ is negative attained at negative value of x, then :

(a) $a > 0$ (b) $b > 0$ (c) $c > 0$ (d) $D > 0$

(where D is discriminant)

11. The quadratic expression $ax^2 + bx + c > 0 \ \forall \ x \in R$, then:

(a) $13a - 5b + 2c > 0$ (b) $13a - b + 2c > 0$

(c) $c > 0,\ D < 0$ (d) $a + c > b,\ D < 0$

(where D is discriminant)

12. The possible positive integral value of 'k' for which $5x^2 - 2kx + 1 < 0$ has exactly one integral solution may be divisible by :

(a) 2 (b) 3 (c) 5 (d) 7

13. If the equation $x^2 + px + q = 0$, the coefficient of x was incorrectly written as 17 instead of 13. Then roots were found to be -2 and -15. The correct roots are :

(a) -1 (b) -3 (c) -5 (d) -10

14. If $x^2 - 3x + 2 > 0$ and $x^2 - 3x - 4 \le 0$, then :

(a) $|x| \le 2$ (b) $2 \le x \le 4$ (c) $-1 \le x < 1$ (d) $2 < x \le 4$

15. If $5^x + (2\sqrt{3})^{2x} - 169 \le 0$ is true for x lying in the interval :

(a) $(-\infty, 2)$ (b) $(0, 2]$ (c) $(2, \infty)$ (d) $(0, 4)$

16. Let $f(x) = x^2 + ax + b$ and $g(x) = x^2 + cx + d$ be two quadratic polynomials with real coefficients and satisfy $ac = 2(b + d)$. Then which of the following is (are) correct?

(a) Exactly one of either $f(x) = 0$ or $g(x) = 0$ must have real roots.

(b) Atleast one of either $f(x) = 0$ or $g(x) = 0$ must have real roots.

(c) Both $f(x) = 0$ and $g(x) = 0$ must have real roots.

(d) Both $f(x) = 0$ and $g(x) = 0$ must have imaginary roots.

17. The expression $\dfrac{1}{\sqrt{x + 2\sqrt{x-1}}} + \dfrac{1}{\sqrt{x - 2\sqrt{x-1}}}$ simplifies to :

(a) $\dfrac{2}{3-x}$ if $1 < x < 2$ (b) $\dfrac{2}{2-x}$ if $1 < x < 2$

(c) $\dfrac{2\sqrt{x-1}}{(x-2)}$ if $x > 2$ (d) $\dfrac{2\sqrt{x-1}}{x+2}$ if $x > 2$

18. If all values of x which satisfies the inequality $\log_{(1/3)}(x^2 + 2px + p^2 + 1) \ge 0$ also satisfy the inequality $kx^2 + kx - k^2 \le 0$ for all real values of k, then all possible values of p lies in the interval :

(a) $[-1, 1]$ (b) $[0, 1]$ (c) $[0, 2]$ (d) $[-2, 0]$

19. Which of the following statement(s) is/are correct?

(a) The number of quadratic equations having real roots which remain unchanged even after squaring their roots is 3.

(b) The number of solutions of the equation $\tan 2\theta + \tan 3\theta = 0$, in the interval $[0, \pi]$ is equal to 6.

(c) For $x_1, x_2, x_3 > 0$, the minimum value of $\frac{2x_1}{x_2} + \frac{128x_3^2}{x_2^2} + \frac{x_2^3}{4x_1x_3^2}$ equals 24.

(d) The locus of the mid-points of chords of the circle $x^2 + y^2 - 2x - 6y - 1 = 0$, which are passing through origin is $x^2 + y^2 - x - 3y = 0$.

20. If $(a, 0)$ is a point on a diameter inside the circle $x^2 + y^2 = 4$. Then $x^2 - 4x - a^2 = 0$ has :

(a) Exactly one real root in $(-1, 0]$
(b) Exactly one real root in $[2, 5]$
(c) Distinct roots greater than -1
(d) Distinct roots less than 5

21. Let $x^2 - px + q = 0$ where $p \in R$, $q \in R$, $pq \neq 0$ have the roots α, β such that $\alpha + 2\beta = 0$, then:

(a) $2p^2 + q = 0$
(b) $2q^2 + p = 0$
(c) $q < 0$
(d) $q > 0$

22. If a, b, c are rational numbers $(a > b > c > 0)$ and quadratic equation $(a + b - 2c)x^2 + (b + c - 2a)x + (c + a - 2b) = 0$ has a root in the interval $(-1, 0)$ then which of the following statement(s) is/are correct ?

(a) $a + c < 2b$
(b) both roots are rational
(c) $ax^2 + 2bx + c = 0$ have both roots negative
(d) $cx^2 + 2bx + a = 0$ have both roots negative

23. For the quadratic polynomial $f(x) = 4x^2 - 8ax + a$, the statements(s) which hold good is/are:

(a) There is only one integral 'a' for which $f(x)$ is non-negative $\forall\ x \in R$
(b) For $a < 0$, the number zero lies between the zeroes of the polynomial
(c) $f(x) = 0$ has two distinct solutions in $(0, 1)$ for a $\in \left(\frac{1}{7}, \frac{4}{7}\right)$
(d) The minimum value of $f(x)$ for minimum value of a for which $f(x)$ is non-negative $\forall x \in R$ is 0

24. Given a, b, c are three distinct real numbers satisfying the inequality $a - 2b + 4c > 0$ and the equation $ax^2 + bx + c = 0$ has no real roots. Then the possible value(s) of $\frac{4a + 2b + c}{a + 3b + 9c}$ is/are :

(a) 2
(b) -1
(c) 3
(d) $\sqrt{2}$

25. Let $f(x) = x^2 - 4x + c\ \forall x \in R$, where c is a real constant, then which of the following is/are true ?

(a) $f(0) > f(1) > f(2)$
(b) $f(2) > f(3) > f(4)$
(c) $f(1) < f(4) < f(-1)$
(d) $f(0) = f(4) > f(3)$

26. If $0 < a < b < c$ and the roots α, β of the equation $ax^2 + bx + c = 0$ are imaginary, then :

(a) $|\alpha| = |\beta|$
(b) $|\alpha| > 1$
(c) $|\beta| < 1$
(d) $|\alpha| = 1$

27. If x satisfies $|x - 1| + |x - 2| + |x - 3| > 6$, then :

(a) $x \in (-\infty, 1)$
(b) $x \in (-\infty, 0)$
(c) $x \in (4, \infty)$
(d) $(2, \infty)$

28. If both roots of the quadratic equation $ax^2 + x + b - a = 0$ are non real and $b > -1$, then which of the following is/are correct?

(a) $a > 0$ (b) $a < b$ (c) $3a > 2 + 4b$ (d) $3a < 2 + 4b$

29. If a, b are two numbers such that $a^2 + b^2 = 7$ and $a^3 + b^3 = 10$, then :

(a) The greatest value of $|a + b| = 5$ (b) The greatest value of $(a + b)$ is 4

(c) The least value of $(a + b)$ is 1 (d) The least value of $|a + b|$ is 1

30. The number of non-negative integral ordered pair(s) (x, y) for which $(xy - 7)^2 = x^2 + y^2$ holds is greater than or equal to :

(a) 1 (b) 2 (c) 3 (d) 4

31. If α, β, γ and δ are the roots of the equation $x^4 - bx - 3 = 0$; then an equation whose roots are $\frac{\alpha+\beta+\gamma}{\delta^2}, \frac{\alpha+\beta+\delta}{\gamma^2}, \frac{\alpha+\gamma+\delta}{\beta^2}$ and $\frac{\beta+\gamma+\delta}{\alpha^2}$ is :

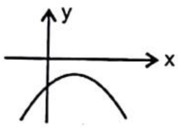
(a) $3x^4 + bx + 1 = 0$ (b) $3x^4 - bx + 1 = 0$

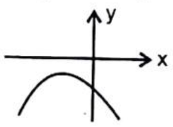
(c) $3x^4 + bx^3 - 1 = 0$ (d) $3x^4 - bx^3 - 1 = 0$

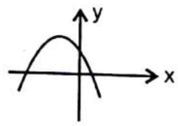
32. The value of k for which both roots of the equation $4x^2 - 2x + k = 0$ are completely in $(-1, 1)$ may be equal to :

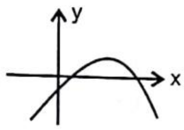
(a) -1 (b) 0 (c) 2 (d) -3

33. If $a, b, c \in R$, then for which of the following graphs of the quadratic polynomial $y = ax^2 - 2bx + c$ $(a \neq 0)$; the product (abc) is negative ?

(a) 

(b) 

(c) 

(d) 

34. If the equation $ax^2 + bx + c = 0$; $a, b, c \in R$ and $a \neq 0$ has no real roots then which of the following is/are always correct ?

(a) $(a + b + c)(a - b + c) > 0$ (b) $(a + b + c)(a - 2b + 4c) > 0$

(c) $(a - b + c)(4a - 2b + c) > 0$ (d) $a(b^2 - 4ac) > 0$

35. If α and β are the roots of the equation $ax^2 + bx + c = 0$; $a, b, c \in R$; $a \neq 0$ then which is (are) correct :

(a) $\alpha^2 + \beta^2 = \frac{b^2 - 2ac}{a^2}$ (b) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{b^2 - 2ac}{c^2}$

(c) $\frac{1}{\alpha^3} + \frac{1}{\beta^3} = \frac{abc - b^3}{c^3}$ (d) $\alpha\beta(\alpha + \beta) = \frac{-bc}{a^2}$

36. The equation $\cos^2 x - \sin x + \lambda = 0$, $x \in (0, \pi/2)$ has roots then value(s) of λ can be equal to :

(a) 0 (b) -1 (c) 1/2 (d) 1

37. If the equation $\ln(x^2 + 5x) - \ln(x + a + 3) = 0$ has exactly one solution for x, then possible integral value of a is :

(a) -3 (b) -1 (c) 0 (d) 2

38. The number of non-negative integral ordered pair(s) (x, y) for which $(xy-7)^2 = x^2 + y^2$ holds is greater than or equal to :

(a) 1 (b) 2 (c) 3 (d) 4

39. If $a < 0$, then the value of x satisfying $x^2 - 2a|x-a| - 3a^2 = 0$ is/are

(a) $a(1-\sqrt{2})$ (b) $a(1+\sqrt{2})$ (c) $a(-1-\sqrt{6})$ (d) $a(-1+\sqrt{6})$

40. If $0 < a < b < c$ and the roots α, β of the equation $ax^2 + bx + c = 0$ are imaginary, then

(a) $|\alpha| = |\beta|$ (b) $|\alpha| > 1$ (c) $|\beta| < 1$ (d) $|\alpha| = 1$

41. If x satisfies $|x-1| + |x-2| + |x-3| > 6$, then

(a) $x \in (-\infty, 1)$ (b) $x \in (-\infty, 0)$ (c) $x \in (4, \infty)$ (d) $(2, \infty)$

42. The value of k for which both roots of the equation $4x^2 - 2x + k = 0$ are completely in $(-1, 1)$, may be equal to :

(a) -1 (b) 0 (c) 2 (d) -3

43. Let $\alpha, \beta, \gamma, \delta$ are roots of $x^4 - 12x^3 + \lambda x^2 - 54x + 14 = 0$

If $\alpha + \beta = \gamma + \delta$, then

(a) $\lambda = 45$ (b) $\lambda = -45$

(c) If $\alpha^2 + \beta^2 < \gamma^2 + \delta^2$ then $\frac{\alpha\beta}{\gamma\delta} = \frac{7}{2}$ (d) If $\alpha^2 + \beta^2 < \gamma^2 + \delta^2 \Rightarrow \frac{\alpha\beta}{\gamma\delta} = \frac{2}{7}$

44. If $\left(\frac{a^3}{a-1}, \frac{a^2-3}{a-1}\right); \left(\frac{b^3}{b-1}, \frac{b^2-3}{b-1}\right); \left(\frac{c^3}{c-1}, \frac{c^2-3}{c-1}\right)$ lie on $L: lx + my + n = 0$; where a, b, c are real numbers different from 1; then

(a) $a + b + c = -\frac{m}{l}$ (b) $abc = \frac{m+n}{l}$

(c) $ab + bc + ca = \frac{n}{l}$ (d) $abc - (ab + bc + ca) + 3(a + b + c) = 0$

Answers

1.	(a, c, d)	**2.**	(a, b, d)	**3.**	(a, c)	**4.**	(a, c, d)	**5.**	(a, b, c, d)	**6.**	(b, c)
7.	(a, b, c)	**8.**	(a, b, d)	**9.**	(a, c)	**10.**	(a, b, d)	**11.**	(a, b, c, d)	**12.**	(a, c)
13.	(b, d)	**14.**	(c, d)	**15.**	(a, b)	**16.**	(b)	**17.**	(b, c)	**18.**	(a, b, c)
19.	(a, b, d)	**20.**	(a, b, c, d)	**21.**	(a, c)	**22.**	(a, b,c, d)	**23.**	(a, b, d)	**24.**	(a, c, d)
25.	(a, c, d)	**26.**	(a, b)	**27.**	(b, c)	**28.**	(a, b)	**29.**	(a, b, d)	**30.**	(a, b, c, d)
31.	(d)	**32.**	(a, b)	**33.**	(a, c, d)	**34.**	(a, b, d)	**35.**	(a, b, d)	**36.**	(a, c)
37.	(b, c, d)	**38.**	(a, b, c, d)	**39.**	(a, d)	**40.**	(a, b)	**41.**	(b, c)	**42.**	(a, b)
43.	(a, c)	**44.**	(a, c, d)								

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

Let $f(x) = ax^2 + bx + c$, $a \neq 0$, such that $f(-1-x) = f(-1+x)\ \forall x \in R$. Also given that $f(x) = 0$ has no real roots and $4a + b > 0$.

1. Let $\alpha = 4a - 2b + c$, $\beta = 9a + 3b + c$, $\gamma = 9a - 3b + c$, then which of the following is correct ?
   (a) $\beta < \alpha < \gamma$ (b) $\gamma < \alpha < \beta$ (c) $\alpha < \gamma < \beta$ (d) $\alpha < \beta < \gamma$
2. Let $p = b - 4a$, $q = 2a + b$, then pq is :
   (a) negative (b) positive (c) 0 (d) nothing can be said

Paragraph for Question Nos. 3 to 4

If α, β are the roots of equation $(k+1)x^2 - (20k+14)x + 91k + 40 = 0; (\alpha < \beta)\, k > 0$, then answer the following questions.

3. The smaller root (α) lie in the interval :
   (a) (4, 7) (b) (7, 10) (c) (10, 13) (d) None of these
4. The larger root (β) lie in the interval :
   (a) (4, 7) (b) (7, 10) (c) (10, 13) (d) None of these

Paragraph for Question Nos. 5 to 7

Let $f(x) = x^2 + bx + c\ \forall x \in R$, $(b, c \in R)$ attains its least value at $x = -1$ and the graph of $f(x)$ cuts y-axis at $y = 2$.

5. The least value of $f(x)\ \forall x \in R$ is :
   (a) -1 (b) 0 (c) 1 (d) 3/2
6. The value of $f(-2) + f(0) + f(1) =$
   (a) 3 (b) 5 (c) 7 (d) 9
7. If $f(x) = a$ has two distinct real roots, then complete set of values of a is :
   (a) $(1, \infty)$ (b) $(-2, -1)$ (c) $(0, 1)$ (d) $(1, 2)$

Paragraph for Question Nos. 8 to 9

Consider the equation $\log_2^2 x - 4\log_2 x - m^2 - 2m - 13 = 0$, $m \in R$. Let the real roots of the equation be x_1, x_2 such that $x_1 < x_2$.

8. The set of all values of m for which the equation has real roots is :
   (a) $(-\infty, 0)$ (b) $(0, \infty)$ (c) $[1, \infty)$ (d) $(-\infty, \infty)$

9. The sum of maximum value of x_1 and minimum value of x_2 is :

(a) $\frac{513}{8}$ (b) $\frac{513}{4}$ (c) $\frac{1025}{8}$ (d) $\frac{257}{4}$

Paragraph for Question Nos. 10 to 11

The equation $x^4 - 2x^3 - 3x^2 + 4x - 1 = 0$ has four distinct real roots x_1, x_2, x_3, x_4 such that $x_1 < x_2 < x_3 < x_4$ and product of two roots is unity, then :

10. $x_1x_2 + x_1x_3 + x_2x_4 + x_3x_4 =$

(a) 0 (b) 1 (c) $\sqrt{5}$ (d) -1

11. $x_2^3 + x_4^3 =$

(a) $\frac{2+5\sqrt{5}}{8}$ (b) -4 (c) $\frac{27\sqrt{5}+5}{4}$ (d) 18

Paragraph for Question Nos. 12 to 14

Let $f(x)$ be a polynomial of degree 5 with leading coefficient unity, such that $f(1) = 5$, $f(2) = 4$, $f(3) = 3$, $f(4) = 2$ and $f(5) = 1$, then :

12. $f(6)$ is equal to :

(a) 120 (b) -120 (c) 0 (d) 6

13. Sum of the roots of $f(x)$ is equal to :

(a) 15 (b) -15 (c) 21 (d) can't be determine

14. Product of the roots of $f(x)$ is equal to :

(a) 120 (b) -120 (c) 114 (d) -114

Paragraph for Question Nos. 15 to 16

Consider the cubic equation in x, $x^3 - x^2 + (x - x^2)\sin\theta + (x - x^2)\cos\theta + (x - 1)\sin\theta\cos\theta = 0$ whose roots are α, β, γ.

15. The value of $\left(\frac{\alpha}{2}\right)^2 + \left(\frac{\beta}{2}\right)^2 + \left(\frac{\gamma}{2}\right)^2 =$

(a) 1 (b) $\frac{1}{2}$

(c) $2\cos\theta$ (d) $\frac{1}{2}(\sin\theta + \cos\theta + \sin\theta\cos\theta)$

16. Number of values of θ in $[0, 2\pi]$ for which at least two roots are equal, is :

(a) 2 (b) 3 (c) 4 (d) 5

Paragraph for Question Nos. 17 to 18

Let $P(x)$ be a quadratic polynomial with real coefficients such that for all real x the relation $2(1+P(x)) = P(x-1) + P(x+1)$ holds.
If $P(0) = 8$ and $P(2) = 32$ then :

17. The sum of all the coefficient of $P(x)$ is :
(a) 20 (b) 19 (c) 17 (d) 15

18. If the range of $P(x)$ is $[m, \infty)$, then the value of m is :
(a) -12 (b) 15 (c) -17 (d) -5

Paragraph for Question Nos. 19 to 21

Let t be a real number satisfying $2t^3 - 9t^2 + 30 - \lambda = 0$ where $t = x + \frac{1}{x}$ and $\lambda \in R$.

19. If the above cubic has three real and distinct solutions for x then exhaustive set of value of λ be :
(a) $3 < \lambda < 10$ (b) $3 < \lambda < 30$ (c) $\lambda = 10$ (d) None of these

20. If the cubic has exactly two real and distinct solutions for x then exhaustive set of values of λ be :
(a) $\lambda \in (-\infty, 3) \cup (30, \infty)$ (b) $\lambda \in (-\infty, -22) \cup (10, \infty) \cup \{3\}$
(c) $\lambda \in \{3, 30\}$ (d) None of these

21. If the cubic has four real and distinct solutions for x then exhaustive set of values of λ be :
(a) $\lambda \in (3, 10)$ (b) $\lambda \in \{3, 10\}$
(c) $\lambda \in (-\infty, -22) \cup (10, \infty)$ (d) None of these

Paragraph for Question Nos. 22 to 23

Consider a quadratic expression $f(x) = tx^2 - (2t-1)x + (5t-1)$

22. If $f(x)$ can take both positive and negative values then t must lie in the interval
(a) $\left(\frac{-1}{4}, \frac{1}{4}\right)$ (b) $\left(-\infty, \frac{-1}{4}\right) \cup \left(\frac{1}{4}, \infty\right)$ (c) $\left(\frac{-1}{4}, \frac{1}{4}\right) - \{0\}$ (d) $(-4, 4)$

23. If $f(x)$ is non-negative $\forall\, x \geq 0$ then t lies in the interval
(a) $\left[\frac{1}{5}, \frac{1}{4}\right]$ (b) $\left[\frac{1}{4}, \infty\right)$ (c) $\left[\frac{-1}{4}, \frac{1}{4}\right]$ (d) $\left[\frac{1}{5}, \infty\right)$

Answers

1.	(c)	**2.**	(a)	**3.**	(a)	**4.**	(c)	**5.**	(c)	**6.**	(d)	**7.**	(a)	**8.**	(d)	**9.**	(d)	**10.**	(b)
11.	(d)	**12.**	(a)	**13.**	(a)	**14.**	(c)	**15.**	(b)	**16.**	(d)	**17.**	(b)	**18.**	(c)	**19.**	(c)	**20.**	(b)
21.	(a)	**22.**	(c)	**23.**	(d)														

Exercise-4 : Matching Type Problems

1.

(A)	The least positive integer x, for which $\frac{2x-1}{2x^3+3x^2+x}$ is positive, is equal to	**(P)**	$\frac{4}{3}$
(B)	If the quadratic equation $3x^2+2(a^2+1)x+(a^2-3a+2)=0$ possess roots of opposite sign then a can be equal to	**(Q)**	1
(C)	The roots of the equation $\sqrt{x+3-4\sqrt{x-1}}+\sqrt{x+8-6\sqrt{x-1}}=1$ can be equal to	**(R)**	6
(D)	If the roots of the equation $x^4-8x^3+bx^2-cx+16=0$ are all real and positive then $2(c-b)$ is equal to	**(S)**	16
		(T)	10

2. Given the inequality $ax+k^2>0$. The complete set of values of 'a' so that

(A)	The inequality is valid for all values of x and k is	**(P)**	R
(B)	There exists a value of x such that the inequality is valid for any value of k is	**(Q)**	ϕ
(C)	There exists a value of k such that the inequality is valid for all values of x is	**(R)**	$\{0\}$
(D)	There exists values of x and k for which inequality is valid is	**(S)**	$R-\{0\}$
		(T)	$\{1\}$

3.

(A)	The real root(s) of the equation $x^4-8x^2-9=0$ are	**(P)**	No real roots
(B)	The real root(s) of the equation $x^{2/3}+x^{1/3}-2=0$ are	**(Q)**	$-3, 3$
(C)	The real root(s) of the equation $\sqrt{3x+1}+1=\sqrt{x}$ are	**(R)**	$-8, 1$

(D)	The real root(s) of the equation $9^x - 10(3^x) + 9 = 0$ are	**(S)**	0, 2

4.

(A)	If a, b are the roots of equation $x^2 + ax + b = 0$ $(a, b \in R)$, then the number of ordered pairs (a, b) is equal to	**(P)**	1
(B)	If $P = \text{cosec}\frac{\pi}{8} + \text{cosec}\frac{2\pi}{8} + \text{cosec}\frac{3\pi}{8} + \text{cosec}\frac{13\pi}{8} + \text{cosec}\frac{14\pi}{8} + \text{cosec}\frac{15\pi}{8}$ and $Q = 8\sin\frac{\pi}{18}\sin\frac{5\pi}{18}\sin\frac{7\pi}{18}$, then $P + Q$ is equal to	**(Q)**	2
(C)	Let $a_1, a_2, a_3 \ldots\ldots$ be positive terms of a G.P. and $a_4, 1, 2, a_{10}$ are the consecutive terms of another G.P. If $\prod_{i=2}^{12} a_i = 4^{\frac{m}{n}}$ where m and n are coprime, then $(m + n)$ equals	**(R)**	3
(D)	For $x, y \in R$, if $x^2 - 2xy + 2y^2 - 6y + 9 = 0$, then the value of $5x - 4y$ is equal to	**(S)**	15

Answers

1. A → Q ; B → P ; C → R ; D → S

2. A → Q ; B → S ; C → R ; D → P

3. A → Q; B → R; C → P; D → S

4. A → Q; B → P; C → S; D → R

Exercise-5 : Subjective Type Problems

1. Let $f(x) = ax^2 + bx + c$ where a, b, c are integers. If $\sin\frac{\pi}{7}\cdot\sin\frac{3\pi}{7} + \sin\frac{3\pi}{7}\cdot\sin\frac{5\pi}{7} + \sin\frac{5\pi}{7}\cdot\sin\frac{\pi}{7} = f\left(\cos\frac{\pi}{7}\right)$, then find the value of $f(2)$:
2. Let a, b, c, d be distinct integers such that the equation $(x-a)(x-b)(x-c)(x-d)-9=0$ has an integer root 'r', then the value of $a+b+c+d-4r$ is equal to :
3. Consider the equation $(x^2+x+1)^2-(m-3)(x^2+x+1)+m=0$, where m is a real parameter. The number of positive integral values of m for which equation has two distinct real roots, is :
4. The number of positive integral values of m, $m \le 16$ for which the equation given in the above questions has 4 distinct real root is :
5. If the equation $(m^2-12)x^4-8x^2-4=0$ has no real roots, then the largest value of m is $p\sqrt{q}$ where p, q are coprime natural numbers, then $p+q=$
6. The least positive integral value of 'x' satisfying $(e^x-2)\left(\sin\left(x+\frac{\pi}{4}\right)\right)(x-\log_e 2)(\sin x-\cos x)<0$ is :
7. The integral values of x for which $x^2+17x+71$ is perfect square of a rational number are a and b, then $|a-b|=$
8. Let $P(x)=x^6-x^5-x^3-x^2-x$ and $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4-x^3-x^2-1=0$, then $P(\alpha)+P(\beta)+P(\gamma)+P(\delta)=$
9. The number of real values of 'a' for which the largest value of the function $f(x)=x^2+ax+2$ in the interval $[-2, 4]$ is 6 will be :
10. The number of all values of n, (where n is a whole number) for which the equation $\frac{x-8}{n-10}=\frac{n}{x}$ has no solution.
11. The number of negative integral values of m for which the expression $x^2+2(m-1)x+m+5$ is positive $\forall\, x>1$ is :
12. If the expression $ax^4+bx^3-x^2+2x+3$ has the remainder $4x+3$ when divided by x^2+x-2, then $a+4b=$.....
13. Find the smallest value of k for which both the roots of equation $x^2-8kx+16(k^2-k+1)=0$ are real, distinct and have values atleast 4.
14. If x^2-3x+2 is a factor of $x^4-px^2+q=0$, then $p+q=$
15. The sum of all real values of k for which the expression $x^2+2xy+ky^2+2x+k=0$ can be resolved into linear factors is :
16. The curve $y=(a+1)x^2+2$ meets the curve $y=ax+3$, $a \ne -1$ in exactly one point, then $a^2=$

17. Find the number of integral values of 'a' for which the range of function $f(x) = \dfrac{x^2 - ax + 1}{x^2 - 3x + 2}$ is $(-\infty, \infty)$.

18. When x^{100} is divided by $x^2 - 3x + 2$, the remainder is $(2^{k+1} - 1)x - 2(2^k - 1)$, then $k =$

19. Let $P(x)$ be a polynomial equation of least possible degree, with rational coefficients, having $\sqrt[3]{7} + \sqrt[3]{49}$ as one of its roots. Then the product of all the roots of $P(x) = 0$ is :

20. The range of values k for which the equation $2\cos^4 x - \sin^4 x + k = 0$ has atleast one solution is $[\lambda, \mu]$. Find the value of $(9\mu + \delta)$.

21. Let $P(x)$ be a polynomial with real coefficient and $P(x) - P'(x) = x^2 + 2x + 1$. Find $P(1)$.

22. Find the smallest positive integral value of a for which the greater root, of the equation $x^2 - (a^2 + a + 1)x + a(a^2 + 1) = 0$ lies between the roots of the equation $x^2 - a^2 x - 2(a^2 - 2) = 0$

23. If the equation $x^4 + kx^2 + k = 0$ has exactly two distinct real roots, then the smallest integral value of $|k|$ is :

24. Let a, b, c, d be the roots of $x^4 - x^3 - x^2 - 1 = 0$. Also consider $P(x) = x^6 - x^5 - x^3 - x^2 - x$, then the value of $P(a) + P(b) + P(c) + P(d)$ is equal to :

25. The number of integral values of a, $a \in [-5, 5]$ for which the equation $x^2 + 2(a-1)x + a + 5 = 0$ has one root smaller than 1 and the other root greater than 3 is :

26. The number of non-negative integral values of n, $n \le 10$ so that a root of the equation $n^2 \sin^2 x - 2\sin x - (2n+1) = 0$ lies in interval $\left[0, \dfrac{\pi}{2}\right]$ is :

27. Let $f(x) = ax^2 + bx + c$, where a, b, c are integers and $a > 1$. If $f(x)$ takes the value p, a prime for two distinct integer values of x, then the number of integer values of x for which $f(x)$ takes the value $2p$ is :

28. If x and y are real numbers connected by the equation $9x^2 + 2xy + y^2 - 92x - 20y + 244 = 0$, then the sum of maximum value of x and the minimum value of y is :

29. Consider two numbers a, b, sum of which is 3 and the sum of their cubes is 7. Then sum of all possible distinct values of a is :

30. If $y^2(y^2 - 6) + x^2 - 8x + 24 = 0$ and the minimum value of $x^2 + y^4$ is m and maximum value is M; then find the value of $M - 2m$.

31. Consider the equation $x^3 - ax^2 + bx - c = 0$, where a, b, c are rational number, $a \ne 1$. It is given that x_1, x_2 and $x_1 x_2$ are the real roots of the equation. If $(b + c) = 2(a + 1)$, then $x_1 x_2 \left(\dfrac{a+1}{b+c}\right) =$

32. Let α satisfy the equation $x^3 + 3x^2 + 4x + 5 = 0$ and β satisfy the equation $x^3 - 3x^2 + 4x - 5 = 0$, $\alpha, \beta \in R$, then $\alpha + \beta =$

33. Let x, y and z are positive reals and $x^2 + xy + y^2 = 2; y^2 + yz + z^2 = 1$ and $z^2 + zx + x^2 = 3$. If the value of $xy + yz + zx$ can be expressed as $\sqrt{\frac{p}{q}}$ where p and q are relatively prime positive integer find the value of $p - q$:

34. The number of ordered pairs (a, b), where a, b are integers satisfying the inequality $\min(x^2 + (a-b)x + (1-a-b)) > \max(-x^2 + (a+b)x - (1+a+b)) \ \forall\ x \in R$, is :

35. The real value of x satisfying $\sqrt[3]{20x + \sqrt[3]{20x+13}} = 13$ can be expressed as $\frac{a}{b}$ where a and b are relatively prime positive integers. Find the value of b ?

36. If the range of the values of a for which the roots of the equation $x^2 - 2x - a^2 + 1 = 0$ lie between the roots of the equation $x^2 - 2(a+1)x + a(a-1) = 0$ is (p, q), then find the value of $\left(q - \frac{1}{p}\right)$.

37. Find the number of positive integers satisfying the inequality $x^2 - 10x + 16 < 0$.

38. If $\sin\theta$ and $\cos\theta$ are the roots of the quadratic equation $ax^2 + bx + c = 0\ (ac \neq 0)$. Then find the value of $\frac{b^2 - a^2}{ac}$.

39. Let the inequality $\sin^2 x + a\cos x + a^2 \geq 1 + \cos x$ is satisfied $\forall\ x \in R$, for $a \in (-\infty, k_1] \cup [k_2, \infty)$, then $|k_1| + |k_2| =$

40. α and β are roots of the equation $2x^2 - 35x + 2 = 0$. Find the value of $\sqrt{(2\alpha - 35)^3 (2\beta - 35)^3}$

41. The sum of all integral values of 'a' for which the equation $2x^2 - (1+2a)x + 1 + a = 0$ has a integral root.

42. Let $f(x)$ be a polynomial of degree 8 such that $F(r) = \frac{1}{r}, r = 1, 2, 3, \ldots, 8, 9$, then $\frac{1}{F(10)} =$

43. Let α, β are two real roots of equation $x^2 + px + q = 0, p, q \in R, q \neq 0$. If the quadratic equation $g(x) = 0$ has two roots $\alpha + \frac{1}{\alpha}, \beta + \frac{1}{\beta}$ such that sum of its roots is equal to product of roots, then then number of integral values q can attain is :

44. If $\cos A$, $\cos B$ and $\cos C$ are the roots of cubic $x^3 + ax^2 + bx + c = 0$, where A, B, C are the angles of a triangle then find the value of $a^2 - 2b - 2c$.

45. Find the number of positive integral values of k for which $kx^2 + (k-3)x + 1 < 0$ for atleast one positive x.

Answers

1.	9	**2.**	0	**3.**	1	**4.**	7	**5.**	5	**6.**	3	**7.**	3	**8.**	6	**9.**	0	**10.**	6
11.	0	**12.**	9	**13.**	2	**14.**	9	**15.**	2	**16.**	4	**17.**	0	**18.**	99	**19.**	56	**20.**	7
21.	2	**22.**	3	**23.**	1	**24.**	6	**25.**	4	**26.**	8	**27.**	0	**28.**	7	**29.**	3	**30.**	4
31.	1	**32.**	0	**33.**	5	**34.**	9	**35.**	5	**36.**	5	**37.**	5	**38.**	2	**39.**	3	**40.**	8
41.	1	**42.**	5	**43.**	3	**44.**	1	**45.**	0										

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Chapter 9 – Sequence and Series

9 SEQUENCE AND SERIES

Exercise-1 : Single Choice Problems

1. If a, b, c are positive numbers and $a+b+c=1$, then the maximum value of $(1-a)(1-b)(1-c)$ is :

(a) 1 (b) $\frac{2}{3}$ (c) $\frac{8}{27}$ (d) $\frac{4}{9}$

2. If $xyz=(1-x)(1-y)(1-z)$ where $0 \le x, y, z \le 1$, then the minimum value of $x(1-z)+y(1-x)+z(1-y)$ is :

(a) $\frac{3}{2}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) $\frac{1}{2}$

3. If $\sec(\alpha-2\beta)$, $\sec\alpha$, $\sec(\alpha+2\beta)$ are in arithmetical progression then $\cos^2\alpha=\lambda\cos^2\beta$ $(\beta \ne n\pi;\ n \in I)$ the value of λ is :

(a) 1 (b) 2 (c) 3 (d) $\frac{1}{2}$

4. Let a, b, c, d, e are non-zero and distinct positive real numbers. If a, b, c are in A.P. ; b, c, d are in G.P. and c, d, e are in H.P., then a, c, e are in :

(a) A.P. (b) G.P.

(c) H.P. (d) Nothing can be said

5. If $(m+1)^{\text{th}}$, $(n+1)^{\text{th}}$, and $(r+1)^{\text{th}}$ terms of a non-constant A.P. are in G.P. and m, n, r are in H.P., then the ratio of first term of the A.P to its common difference is :

(a) $-\frac{n}{2}$ (b) $-n$ (c) $-2n$ (d) $+n$

6. If the equation $x^4-4x^3+ax^2+bx+1=0$ has four positive roots, then the value of $(a+b)$ is :

(a) -4 (b) 2

(c) 6 (d) can not be determined

7. If S_1, S_2 and S_3 are the sums of first n natural numbers, their squares and their cubes respectively, then $\dfrac{S_1^4 S_2^2 - S_2^2 S_3^2}{S_1^2 + S_3^2} =$

(a) 4 (b) 2 (c) 1 (d) 0

8. If $S_n = \dfrac{1\cdot 2}{3!} + \dfrac{2\cdot 2^2}{4!} + \dfrac{3\cdot 2^3}{5!} + \ldots\ldots$ upto n terms then the sum of the infinite terms is :

(a) 1 (b) $\dfrac{2}{3}$ (c) e (d) $\dfrac{\pi}{4}$

9. If $\tan\left(\dfrac{\pi}{12} - x\right)$, $\tan\dfrac{\pi}{12}$, $\tan\left(\dfrac{\pi}{12} + x\right)$ in order are three consecutive terms of a G.P. then sum of all the solutions in [0, 314] is $k\pi$. The value of k is :

(a) 4950 (b) 5050 (c) 2525 (d) 5010

10. Let $S_k = 1 + 2 + 3 + \ldots\ldots + k$ and $Q_n = \dfrac{S_2}{S_2 - 1}\cdot\dfrac{S_3}{S_3 - 1}\cdot\dfrac{S_4}{S_4 - 1}\ldots\ldots\dfrac{S_n}{S_n - 1}$, where $k, n \in N$

$\lim_{n\to\infty} Q_n =$

(a) $\dfrac{1}{3}$ (b) 1 (c) 3 (d) 0

11. l, m, n are the p^{th}, q^{th} and r^{th} term of a G.P. all positive, then $\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$ equals :

(a) −1 (b) 2 (c) 1 (d) 0

12. The number of natural numbers < 300 that are divisible by 6 but not by 9 is :

(a) 49 (b) 37 (c) 33 (d) 16

13. If $x, y, z > 0$ and $x + y + z = 1$ then $\dfrac{xyz}{(1-x)(1-y)(1-z)}$ is necessarily.

(a) ≥ 8 (b) $\leq \dfrac{1}{8}$ (c) 1 (d) None of these

14. If the roots of the equation $px^2 + qx + r = 0$, where $2p, q, 2r$ are in G.P., are of the form $\alpha^2, 4\alpha - 4$. Then the value of $2p + 4q + 7r$ is :

(a) 0 (b) 10 (c) 14 (d) 18

15. Let $x_1, x_2, x_3, \ldots\ldots, x_k$ be the divisors of positive integer n (including 1 and n). If $x_1 + x_2 + x_3 + \ldots\ldots + x_k = 75$. Then $\sum_{i=1}^{k}\left(\dfrac{1}{x_i}\right)$ is equal to :

(a) $\dfrac{75}{k}$ (b) $\dfrac{75}{n}$ (c) $\dfrac{1}{n}$ (d) $\dfrac{1}{75}$

16. If $a_1, a_2, a_3, \ldots\ldots, a_n$ are in H.P. and $f(k) = \sum_{r=1}^{n} a_r - a_k$ then $\frac{a_1}{f(1)}, \frac{a_2}{f(2)}, \frac{a_3}{f(3)}, \ldots\ldots, \frac{a_n}{f(n)}$ are in:

(a) A.P. (b) G.P. (c) H.P. (d) None of these

17. If α, β be roots of the equation $375x^2 - 25x - 2 = 0$ and $s_n = \alpha^n + \beta^n$, then $\lim_{n \to \infty}\left(\sum_{r=1}^{n} S_r\right) = \ldots\ldots$

(a) $\frac{1}{12}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) 1

18. If a_i, $i = 1, 2, 3, 4$ be four real members of the same sign, then the minimum value of $\sum \frac{a_i}{a_j}$, $i, j \in \{1, 2, 3, 4\}, i \neq j$ is :

(a) 6 (b) 8 (c) 12 (d) 24

19. Given that x, y, z are positive reals such that $xyz = 32$. The minimum value of $x^2 + 4xy + 4y^2 + 2z^2$ is equal to :

(a) 64 (b) 256 (c) 96 (d) 216

20. In an A.P., five times the fifth term is equal to eight times the eighth term. Then the sum of the first twenty five terms is equal to :

(a) 25 (b) $\frac{25}{2}$ (c) -25 (d) 0

21. Let α, β be two distinct values of x lying in $[0, \pi]$ for which $\sqrt{5}\sin x$, $10\sin x$, $10(4\sin^2 x + 1)$ are 3 consecutive terms of a G.P. Then minimum value of $|\alpha - \beta| =$

(a) $\frac{\pi}{10}$ (b) $\frac{\pi}{5}$ (c) $\frac{2\pi}{5}$ (d) $\frac{3\pi}{5}$

22. In an infinite G.P., the sum of first three terms is 70. If the extreme terms are multiplied by 4 and the middle term is multiplied by 5, the resulting terms form an A.P. then the sum to infinite terms of G.P. is :

(a) 120 (b) 40 (c) 160 (d) 80

23. The value of the sum $\sum_{k=1}^{\infty}\sum_{n=1}^{\infty} \frac{k}{2^{n+k}}$ is equal to :

(a) 5 (b) 4 (c) 3 (d) 2

24. Let p, q, r are positive real numbers, such that $27pqr \geq (p + q + r)^3$ and $3p + 4q + 5r = 12$, then $p^3 + q^4 + r^5 =$

(a) 3 (b) 6 (c) 2 (d) 4

25. Find the sum of the infinite series $\frac{1}{9} + \frac{1}{18} + \frac{1}{30} + \frac{1}{45} + \frac{1}{63} + \ldots\ldots$

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{5}$ (d) $\frac{2}{3}$

26. If S_r denote the sum of first 'r' terms of a non constant A.P. and $\frac{S_a}{a^2}=\frac{S_b}{b^2}=c$, where a, b, c are distinct then S_c =

(a) c^2 (b) c^3 (c) c^4 (d) abc

27. In an infinite G.P. second term is x and its sum is 4, then complete set of values of 'x' is :

(a) $(-8, 0)$ (b) $\left[-\frac{1}{8}, \frac{1}{8}\right)-\{0\}$

(c) $\left[-1, -\frac{1}{8}\right)\cup\left(\frac{1}{8}, 1\right]$ (d) $(-8, 1]-\{0\}$

28. The number of terms of an A.P. is odd. The sum of the odd terms (1^{st}, 3^{rd} etc.,) is 248 and the sum of the even terms is 217. The last term exceeds the first by 56, then :

(a) the number of terms is 17 (b) the first term is 3

(c) the number of terms is 13 (d) the first term is 1

29. Let $A_1, A_2, A_3, \ldots\ldots, A_n$ be squares such that for each $n \geq 1$ the length of a side of A_n equals the length of a diagonal of A_{n+1}. If the side of A_1 be 20 units then the smallest value of 'n' for which area of A_n is less than 1.

(a) 7 (b) 8 (c) 9 (d) 10

30. Let $S_k=\sum_{i=0}^{\infty}\frac{1}{(k+1)^i}$, then $\sum_{k=1}^{n} kS_k$ equal :

(a) $\frac{n(n+1)}{2}$ (b) $\frac{n(n-1)}{2}$ (c) $\frac{n(n+2)}{2}$ (d) $\frac{n(n+3)}{2}$

31. The sum of the series $\frac{2}{1\cdot 2}+\frac{5}{2\cdot 3}2^1+\frac{10}{3\cdot 4}2^2+\frac{17}{4\cdot 5}2^3+\ldots\ldots$ upto n terms is equal :

(a) $\frac{n2^n}{n+1}$ (b) $\left(\frac{n}{n+1}\right)2^n+1$ (c) $\frac{n2^n}{n+1}-1$ (d) $\frac{(n-1)2^n}{n+1}$

32. If $(1\cdot 5)^{30}=k$, then the value of $\sum_{n=2}^{29}(1\cdot 5)^n$, is :

(a) $2k-3$ (b) $k+1$ (c) $2k+7$ (d) $2k-\frac{9}{2}$

33. n arithmetic means are inserted between 7 and 49 and their sum is found to be 364, then n is :

(a) 11 (b) 12 (c) 13 (d) 14

34. The third term of a G.P. is 2. Then the product of the first five terms, is :

(a) 2^3 (b) 2^4 (c) 2^5 (d) none of these

35. The sum of first n terms of an A.P. is $5n^2+4n$, its common difference is :

(a) 9 (b) 10 (c) 3 (d) -4

36. If $x + y = a$ and $x^2 + y^2 = b$, then the value of $(x^3 + y^3)$, is :

(a) ab (b) $a^2 + b$ (c) $a + b^2$ (d) $\frac{3ab - a^3}{2}$

37. If $S_1, S_2, S_3, \ldots\ldots, S_n$ are the sum of infinite geometric series whose first terms are $1, 3, 5, \ldots\ldots, (2n-1)$ and whose common ratios are $\frac{2}{3}, \frac{2}{5}, \ldots\ldots, \frac{2}{2n+1}$ respectively, then

$\left\{\frac{1}{S_1 S_2 S_3} + \frac{1}{S_2 S_3 S_4} + \frac{1}{S_3 S_4 S_5} + \ldots\ldots \text{upto infinite terms}\right\} =$

(a) $\frac{1}{15}$ (b) $\frac{1}{60}$ (c) $\frac{1}{12}$ (d) $\frac{1}{3}$

38. Sequence $\{t_n\}$ of positive terms is a G.P. If t_6, 2, 5, t_{14} form another G.P. in that order, then the product $t_1 t_2 t_3 \ldots\ldots t_{18} t_{19}$ is equal to :

(a) 10^9 (b) 10^{10} (c) $10^{17/2}$ (d) $10^{19/2}$

39. The minimum value of $\frac{(A^2 + A + 1)(B^2 + B + 1)(C^2 + C + 1)(D^2 + D + 1)}{ABCD}$ where $A, B, C, D > 0$ is :

(a) $\frac{1}{3^4}$ (b) $\frac{1}{2^4}$ (c) 2^4 (d) 3^4

40. If $\sum_1^{20} r^3 = a$, $\sum_1^{20} r^2 = b$ then sum of products of 1, 2, 3, 4...... 20 taking two at a time is :

(a) $\frac{a-b}{2}$ (b) $\frac{a^2 - b^2}{2}$ (c) $a - b$ (d) $a^2 - b^2$

41. The sum of the first $2n$ terms of an A.P. is x and the sum of the next n terms is y, its common difference is :

(a) $\frac{x - 2y}{3n^2}$ (b) $\frac{2y - x}{3n^2}$ (c) $\frac{x - 2y}{3n}$ (d) $\frac{2y - x}{3n}$

42. The number of non-negative integers 'n' satisfying $n^2 = p + q$ and $n^3 = p^2 + q^2$ where p and q are integers.

(a) 2 (b) 3 (c) 4 (d) Infinite

43. Concentric circles of radii 1, 2, 3 ...... 100 cms are drawn. The interior of the smallest circle is coloured red and the angular regions are coloured alternately green and red, so that no two adjacent regions are of the same colour. The total area of the green regions in sq. cm is equals to:

(a) 1000π (b) 5050π (c) 4950π (d) 5151π

44. If $\log_2 4$, $\log_{\sqrt{2}} 8$ and $\log_3 9^{k-1}$ are consecutive terms of a geometric sequence, then the number of integers that satisfy the system of inequalities $x^2 - x > 6$ and $|x| < k^2$ is :

(a) 193 (b) 194 (c) 195 (d) 196

45. Let T_r be the r^{th} term of an A.P. whose first term is $-\frac{1}{2}$ and common difference is 1, then $\sum_{r=1}^{n}\sqrt{1+T_rT_{r+1}T_{r+2}T_{r+3}} =$

(a) $\frac{n(n+1)(2n+1)}{6}-\frac{5n}{4}$ (b) $\frac{n(n+1)(2n+1)}{6}-\frac{5n}{4}+\frac{1}{4}$

(c) $\frac{n(n+1)(2n+1)}{6}-\frac{5n}{4}+\frac{1}{2}$ (d) $\frac{n(n+1)(2n+1)}{12}-\frac{5n}{8}+1$

46. If $\sum_{r=1}^{n} T_r = \frac{n(n+1)(n+2)}{3}$, then $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{2008}{T_r} =$

(a) 2008 (b) 3012 (c) 4016 (d) 8032

47. The sum of the infinite series, $1^2-\frac{2^2}{5}+\frac{3^2}{5^2}-\frac{4^2}{5^3}+\frac{5^2}{5^4}-\frac{6^2}{5^5}+.....$ is :

(a) $\frac{1}{2}$ (b) $\frac{25}{24}$ (c) $\frac{25}{54}$ (d) $\frac{125}{252}$

48. The absolute term in $P(x)=\sum_{r=1}^{n}\left(x-\frac{1}{r}\right)\left(x-\frac{1}{r+1}\right)\left(x-\frac{1}{r+2}\right)$ as n approaches to infinity is :

(a) $\frac{1}{2}$ (b) $\frac{-1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{-1}{4}$

49. Let a, b, c are positive real numbers such that $p=a^2b+ab^2-a^2c-ac^2$; $q=b^2c+bc^2-a^2b-ab^2$ and $r=ac^2+a^2c-cb^2-bc^2$ and the quadratic equation $px^2+qx+r=0$ has equal roots ; then a, b, c are in :

(a) A.P. (b) G.P. (c) H.P. (d) None of these

50. If T_k denotes the k^{th} term of an H.P. from the begining and $\frac{T_2}{T_6}=9$, then $\frac{T_{10}}{T_4}$ equals :

(a) $\frac{17}{5}$ (b) $\frac{5}{17}$ (c) $\frac{7}{19}$ (d) $\frac{19}{7}$

51. Number of terms common to the two sequences 17, 21, 25, ..., 417 and 16, 21, 26, ..., 466 is :

(a) 19 (b) 20 (c) 21 (d) 22

52. The sum of the series $1+\frac{2}{3}+\frac{1}{3^2}+\frac{2}{3^3}+\frac{1}{3^4}+\frac{2}{3^5}+\frac{1}{3^6}+\frac{2}{3^7}+......$ upto infinite terms is equal to :

(a) $\frac{15}{8}$ (b) $\frac{8}{15}$ (c) $\frac{27}{8}$ (d) $\frac{21}{8}$

53. The coefficient of x^8 in the polynomial $(x-1)(x-2)(x-3)......(x-10)$ is :

(a) 2640 (b) 1320 (c) 1370 (d) 2740

54. Let $\alpha = \lim_{n\to\infty} \frac{(1^3 - 1^2) + (2^3 - 2^2) + \ldots + (n^3 - n^2)}{n^4}$, then α is equal to :

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) non-existent

55. If $16x^4 - 32x^3 + ax^2 + bx + 1 = 0$, $a, b \in R$ has positive real roots only, then $a - b$ is equal to :

(a) -32 (b) 32 (c) 49 (d) -49

56. If ABC is a triangle and $\tan\frac{A}{2}$, $\tan\frac{B}{2}$, $\tan\frac{C}{2}$ are in H.P., then the minimum value of $\cot\frac{B}{2} =$

(a) $\sqrt{3}$ (b) 1 (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$

57. If α and β are the roots of the quadratic equation $4x^2 + 2x - 1 = 0$ then the value of $\sum_{r=1}^{\infty}(\alpha^r + \beta^r)$ is :

(a) 2 (b) 3 (c) 6 (d) 0

58. The sum of the series $2^2 + 2(4)^2 + 3(6)^2 + \ldots\ldots$ upto 10 terms is equal to :

(a) 11300 (b) 12100 (c) 12300 (d) 11200

59. If a and b are positive real numbers such that $a + b = 6$, then the minimum value of $\left(\frac{4}{a} + \frac{1}{b}\right)$ is equal to :

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) 1 (d) $\frac{3}{2}$

60. The first term of an infinite G.P. is the value of x satisfying the equation $\log_4(4^x - 15) + x - 2 = 0$ and the common ratio is $\cos\left(\frac{2011\pi}{3}\right)$. The sum of G.P. is :

(a) 1 (b) $\frac{4}{3}$ (c) 4 (d) 2

61. Let a, b, c be positive numbers, then the minimum value of $\frac{a^4 + b^4 + c^2}{abc}$ is :

(a) 4 (b) $2^{3/4}$ (c) $\sqrt{2}$ (d) $2\sqrt{2}$

62. If $xy = 1$; then minimum value of $x^2 + y^2$ is :

(a) 1 (b) 2 (c) $\sqrt{2}$ (d) 4

63. Find the value of $\frac{2}{1^3} + \frac{6}{1^3 + 2^3} + \frac{12}{1^3 + 2^3 + 3^3} + \frac{20}{1^3 + 2^3 + 3^3 + 4^3} + \ldots$ upto 60 terms :

(a) 2 (b) $\frac{1}{2}$ (c) 4 (d) $\frac{1}{4}$

64. Evaluate : $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)(n+3)....(n+k)}$

(a) $\frac{1}{(k-1)(k-1)!}$ (b) $\frac{1}{k \cdot k!}$ (c) $\frac{1}{(k-1)k!}$ (d) $\frac{1}{k!}$

65. Consider two positive numbers a and b. If arithmetic mean of a and b exceeds their geometric mean by 3/2 and geometric mean of a and b exceeds their harmonic mean by 6/5 then the value of $a^2 + b^2$ will be :

(a) 150 (b) 153 (c) 156 (d) 159

66. Sum of first 10 terms of the series, $S = \frac{7}{2^2 \cdot 5^2} + \frac{13}{5^2 \cdot 8^2} + \frac{19}{8^2 \cdot 11^2} + ...$ is :

(a) $\frac{255}{1024}$ (b) $\frac{88}{1024}$ (c) $\frac{264}{1024}$ (d) $\frac{85}{1024}$

67. $\sum_{r=1}^{10} \frac{r}{1-3r^2+r^4} =$

(a) $-\frac{50}{109}$ (b) $-\frac{54}{109}$ (c) $-\frac{55}{111}$ (d) $-\frac{55}{109}$

68. Let r^{th} term t_r of a series is given by $t_r = \frac{r}{1+r^2+r^4}$. Then $\lim_{n\to\infty} \sum_{r=1}^{n} t_r$ is equal to :

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) $\frac{1}{4}$

69. The sum of the series $1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + ...$ to infinite terms, is :

(a) $\frac{31}{12}$ (b) $\frac{41}{16}$ (c) $\frac{45}{16}$ (d) $\frac{35}{16}$

70. The third term of a G.P. is 2. Then the product of the first five terms, is :

(a) 2^3 (b) 2^4 (c) 2^5 (d) none of these

71. If $x_1, x_2, x_3, ... x_{2n}$ are in A.P., then $\sum_{r=1}^{2n} (-1)^{r+1} x_r^2$ is equal to :

(a) $\frac{n}{(2n-1)}(x_1^2 - x_{2n}^2)$ (b) $\frac{2n}{(2n-1)}(x_1^2 - x_{2n}^2)$

(c) $\frac{n}{n-1}(x_1^2 - x_{2n}^2)$ (d) $\frac{n}{2n+1}(x_1^2 - x_{2n}^2)$

72. Let two numbers have arithmatic mean 9 and geometric mean 4. Then these numbers are roots of the equation :

(a) $x^2 + 18x + 16 = 0$ (b) $x^2 - 18x - 16 = 0$

(c) $x^2 + 18x - 16 = 0$ (d) $x^2 - 18x + 16 = 0$

73. If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of $(p+q)$ is :

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

74. A person has to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = \ldots = a_{10} = 150$ and $a_{10}, a_{11}, a_{12}, \ldots$ are in A.P. with common difference -2, then the time taken by him to count all notes is :

(a) 34 minutes (b) 24 minutes (c) 125 minutes (d) 35 minutes

75. A non constant arithmatic progression has common difference d and first term is $(1 - ad)$. If the sum of the first 20 terms is 20, then the value of a is equal to :

(a) $\frac{2}{19}$ (b) $\frac{19}{2}$ (c) $\frac{2}{9}$ (d) $\frac{9}{2}$

76. The value of $\sum_{n=3}^{\infty} \frac{1}{n^5 - 5n^3 + 4n} =$

(a) $\frac{1}{120}$ (b) $\frac{1}{96}$ (c) $\frac{1}{24}$ (d) $\frac{1}{144}$

77. Find the value of $\frac{2}{1^3} + \frac{6}{1^3 + 2^3} + \frac{12}{1^3 + 2^3 + 3^3} + \frac{20}{1^3 + 2^3 + 3^3 + 4^3} + \ldots$ up to infinite terms:

(a) 2 (b) $\frac{1}{2}$ (c) 4 (d) $\frac{1}{4}$

78. The minimum value of the expression $2^x + 2^{2x+1} + \frac{5}{2^x}, x \in R$ is :

(a) 7 (b) $(7.2)^{1/7}$ (c) 8 (d) $(3.10)^{1/3}$

79. The value of $\sum_{r=1}^{\infty} \frac{(4r+5)5^{-r}}{r(5r+5)}$ is :

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{1}{25}$ (d) $\frac{2}{25}$

Answers

1.	(c)	**2.**	(c)	**3.**	(b)	**4.**	(b)	**5.**	(a)	**6.**	(b)	**7.**	(d)	**8.**	(a)	**9.**	(a)	**10.**	(c)
11.	(d)	**12.**	(c)	**13.**	(b)	**14.**	(c)	**15.**	(b)	**16.**	(c)	**17.**	(a)	**18.**	(c)	**19.**	(c)	**20.**	(d)
21.	(b)	**22.**	(d)	**23.**	(d)	**24.**	(a)	**25.**	(a)	**26.**	(b)	**27.**	(d)	**28.**	(b)	**29.**	(d)	**30.**	(d)
31.	(a)	**32.**	(d)	**33.**	(c)	**34.**	(c)	**35.**	(b)	**36.**	(d)	**37.**	(b)	**38.**	(d)	**39.**	(d)	**40.**	(a)
41.	(b)	**42.**	(b)	**43.**	(b)	**44.**	(a)	**45.**	(c)	**46.**	(a)	**47.**	(c)	**48.**	(d)	**49.**	(c)	**50.**	(b)
51.	(b)	**52.**	(a)	**53.**	(b)	**54.**	(b)	**55.**	(b)	**56.**	(a)	**57.**	(d)	**58.**	(b)	**59.**	(d)	**60.**	(c)
61.	(d)	**62.**	(b)	**63.**	(c)	**64.**	(c)	**65.**	(d)	**66.**	(d)	**67.**	(d)	**68.**	(a)	**69.**	(d)	**70.**	(c)
71.	(a)	**72.**	(d)	**73.**	(d)	**74.**	(a)	**75.**	(b)	**76.**	(b)	**77.**	(c)	**78.**	(c)	**79.**	(a)		

Exercise-2 : One or More than One Answer is/are Correct

1. If the first and $(2n-1)^{th}$ terms of an A.P., G.P. and H.P. with positive terms are equal and their n^{th} terms are a, b and c respectively, then which of the following options must be correct :

(a) $a+c=2b$ (b) $a \geq b \geq c$

(c) $\frac{2ac}{a+c}=b$ (d) $ac=b^2$

2. Let a, b, c are distinct real numbers such that expression ax^2+bx+c, bx^2+cx+a and cx^2+ax+b are always positive then possible value(s) of $\frac{a^2+b^2+c^2}{ab+bc+ca}$ may be :

(a) 1 (b) 2 (c) 3 (d) 4

3. If a, b, c are in H.P., where $a > c > 0$, then :

(a) $b > \frac{a+c}{2}$ (b) $\frac{1}{a-b}-\frac{1}{b-c}<0$

(c) $ac > b^2$ (d) $bc(1-a), ac(1-b), ab(1-c)$ are in A.P.

4. In an A.P., let T_r denote r^{th} term from beginning, $T_p=\frac{1}{q(p+q)}$, $T_q=\frac{1}{p(p+q)}$, then :

(a) T_1 = common difference (b) $T_{p+q}=\frac{1}{pq}$

(c) $T_{pq}=\frac{1}{p+q}$ (d) $T_{p+q}=\frac{1}{p^2q^2}$

5. Which of the following statement(s) is(are) correct?

(a) Sum of the reciprocal of all the n harmonic means inserted between a and b is equal to n times the harmonic mean between two given numbers a and b.

(b) Sum of the cubes of first n natural number is equal to square of the sum of the first n natural numbers.

(c) If $a, A_1, A_2, A_3, \ldots\ldots, A_{2n}, b$ are in A.P. then $\sum_{i=1}^{2n} A_i = n(a+b)$.

(d) If the first term of the geometric progression $g_1, g_2, g_3, \ldots\ldots, \infty$ is unity, then the value of the common ratio of the progression such that $(4g_2+5g_3)$ is minimum equals $\frac{2}{5}$.

6. If a, b, c are in 3 distinct numbers in H.P., $a, b, c > 0$, then :

(a) $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in A.P. (b) $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in A.P.

(c) $a^5+c^5 \geq 2b^5$ (d) $\frac{a-b}{b-c}=\frac{a}{c}$

7. All roots of equation $x^5 - 40x^4 + \alpha x^3 + \beta x^2 + \gamma x + \delta = 0$ are in G.P. If the sum of their reciprocals is 10, then δ can be equal to :

(a) 32 (b) -32 (c) $\frac{1}{32}$ (d) $-\frac{1}{32}$

8. Let $a_1, a_2, a_3, ...$ be a sequence of non-zero real numbers which are in A.P. for $k \in N$. Let $f_k(x) = a_k x^2 + 2a_{k+1}x + a_{k+2}$

(a) $f_k(x) = 0$ has real roots for each $k \in N$.
(b) Each of $f_k(x) = 0$ has one root in common.
(c) Non-common roots of $f_1(x) = 0, f_2(x) = 0, f_3(x) = 0, ...$ from an A.P.
(d) None of these

9. Given a, b, c are in A.P., b, c, d are in G.P. and c, d, e are in H.P. If $a = 2$ and $e = 18$, then the possible value of 'c' can be :

(a) 9 (b) -6 (c) 6 (d) -9

10. The number a, b, c in that order form a three term A.P and $a + b + c = 60$. The number $(a-2)$, b, $(c+3)$ in that order form a three term G.P. All possible values of $(a^2 + b^2 + c^2)$ is/are :

(a) 1218 (b) 1208 (c) 1288 (d) 1298

11. If $(x^2 + x + 1) + (x^2 + 2x + 3) + (x^2 + 3x + 5) + ... + (x^2 + 20x + 39) = 4500$, then x is equal to :

(a) 10 (b) -10 (c) 20.5 (d) -20.5

12. For ΔABC, if $81 + 144a^4 + 16b^4 + 9c^4 = 144abc$, (where notations have their usual meaning), then :

(a) $a > b > c$ (b) $A < B < C$
(c) Area of $\Delta ABC = \frac{3\sqrt{3}}{8}$ (d) Triangle ABC is right angled

13. Let $x, y, z \in \left(0, \frac{\pi}{2}\right)$ are first three consecutive terms of an arithmatic progression such that $\cos x + \cos y + \cos z = 1$ and $\sin x + \sin y + \sin z = \frac{1}{\sqrt{2}}$, then which of the following is/are correct ?

(a) $\cot y = \sqrt{2}$ (b) $\cos(x-y) = \frac{\sqrt{3}-\sqrt{2}}{2\sqrt{2}}$
(c) $\tan 2y = \frac{2\sqrt{2}}{3}$ (d) $\sin(x-y) + \sin(y-z) = 0$

14. If the numbers 16, 20, 16, d form a A.G.P., then d can be equal to.

(a) 3 (b) 11 (c) -8 (d) -16

15. Given $\dfrac{\underbrace{1000.....01}_{n\text{ zeroes}}}{\underbrace{1000.....01}_{(n+1)\text{ zeroes}}} < \dfrac{\underbrace{1000.....01}_{m\text{ zeroes}}}{\underbrace{1000.....01}_{(m+1)\text{ zeroes}}}$, then which of the following is true

(a) $m+1<n$ (b) $m<n$ (c) $m<n+1$ (d) $m>n+1$

16. If $S_r = \sqrt{r+\sqrt{r+\sqrt{r+\sqrt{r+\sqrt{....\infty}}}}}$, $r>0$, then which of the following is/are correct.

(a) S_2, S_6, S_{12}, S_{20} are in A.P. (b) S_4, S_9, S_{16} are irrational

(c) $(2S_3-1)^2, (2S_4-1)^2, (2S_5-1)^2$ are in A.P. (d) S_2, S_{12}, S_{56} are in G.P.

17. Consider the A.P. 50, 48, 46, 44, ... If S_n denotes the sum to n terms of this A.P., then

(a) S_n is maximum for $n = 25$ (b) the first negative terms is 26^{th} term

(c) the first negative term is 27^{th} term (d) the maximum value of S_n is 650

18. Let S_n be the sum to n terms of the series $\dfrac{3}{1^2}+\dfrac{5}{1^2+2^2}+\dfrac{7}{1^2+2^2+3^2}+\dfrac{9}{1^2+2^2+3^2+4^2}$ +...... then

(a) $S_5 = 5$ (b) $S_{50} = \dfrac{100}{17}$ (c) $S_{1001} = \dfrac{1001}{97}$ (d) $S_\infty = 6$

19. For ΔABC, if $81+144a^4+16b^4+9c^4 = 144abc$, (where notations have their usual meaning), then

(a) $a>b>c$ (b) $A<B<C$

(c) Area of $\Delta ABC = \dfrac{3\sqrt{3}}{8}$ (d) Triangle ABC is right angled

Answers

1.	(b, d)	2.	(b, c)	3.	(b, c, d)	4.	(a, b, c)	5.	(b, c)	6.	(a, b, c, d)
7.	(a, b)	8.	(a, b)	9.	(b, c)	10.	(b, d)	11.	(a, d)	12.	(b, c, d)
13.	(a, b)	14.	(b)	15.	(b, c)	16.	(a, b, c, d)	17.	(a, c, d)	18.	(a, b, d)
19.	(b, c, d)										

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

The first four terms of a sequence are given by $T_1 = 0, T_2 = 1, T_3 = 1, T_4 = 2$.
The general term is given by $T_n = A\alpha^{n-1} + B\beta^{n-1}$ where A, B, α, β are independent of n and A is positive.

1. The value of $(\alpha^2 + \beta^2 + \alpha\beta)$ is equal to :
   (a) 1 (b) 2 (c) 5 (d) 4
2. The value of $5(A^2 + B^2)$ is equal to :
   (a) 2 (b) 4 (c) 6 (d) 8

Paragraph for Question Nos. 3 to 4

There are two sets A and B each of which consists of three numbers in A.P. whose sum is 15. D and d are their respective common differences such that $D - d = 1$, $D > 0$. If $\frac{p}{q} = \frac{7}{8}$ where p and q are the product of the numbers in those sets A and B respectively.

3. Sum of the product of the numbers in set A taken two at a time is :
   (a) 51 (b) 71 (c) 74 (d) 86
4. Sum of the product of the numbers in set B taken two at a time is :
   (a) 52 (b) 54 (c) 64 (d) 74

Paragraph for Question Nos. 5 to 7

Let x, y, z are positive reals and $x + y + z = 60$ and $x > 3$.

5. Maximum value of $(x-3)(y+1)(z+5)$ is :
   (a) (17) (21) (25) (b) (20) (21) (23) (c) (21) (21) (21) (d) (23) (19) (15)
6. Maximum value of $(x-3)(2y+1)(3z+5)$ is :
   (a) $\frac{(355)^3}{3^3 \cdot 6^2}$ (b) $\frac{(355)^3}{3^3 \cdot 6^3}$ (c) $\frac{(355)^3}{3^2 \cdot 6^3}$ (d) None of these
7. Maximum value of xyz is :
   (a) 8×10^3 (b) 27×10^3 (c) 64×10^3 (d) 125×10^3

Paragraph for Question Nos. 8 to 10

Two consecutive numbers from n natural numbers 1, 2, 3, ..., n are removed. Arithmetic mean of the remaining numbers is $\frac{105}{4}$.

8. The value of n is :

(a) 48 (b) 50 (c) 52 (d) 49

9. The G.M. of the removed numbers is :

(a) $\sqrt{30}$ (b) $\sqrt{42}$ (c) $\sqrt{56}$ (d) $\sqrt{72}$

10. Let removed numbers are x_1, x_2 then $x_1 + x_2 + n =$

(a) 61 (b) 63 (c) 65 (d) 69

Paragraph for Question Nos. 11 to 13

The sequence $\{a_n\}$ is defined by formula $a_0 = 4$ and $a_{n+1} = a_n^2 - 2a_n + 2$ for $n \geq 0$. Let the sequence $\{b_n\}$ is defined by formula $b_0 = \frac{1}{2}$ and $b_n = \frac{2a_0 a_1 a_2 \ldots\ldots a_{n-1}}{a_n}$ $\forall\ n \geq 1$.

11. The value of a_{10} is equal to :

(a) $1 + 2^{1024}$ (b) 4^{1024} (c) $1 + 3^{1024}$ (d) 6^{1024}

12. The value of n for which $b_n = \frac{3280}{3281}$ is :

(a) 2 (b) 3 (c) 4 (d) 5

13. The sequence $\{b_n\}$ satisfies the recurrence formula :

(a) $b_{n+1} = \frac{2b_n}{1 - b_n^2}$ (b) $b_{n+1} = \frac{2b_n}{1 + b_n^2}$

(c) $\frac{b_n}{1 + 2b_n^2}$ (d) $\frac{b_n}{1 - 2b_n^2}$

Paragraph for Question Nos. 14 to 15

Let $f(n) = \sum_{r=2}^{n} \frac{r}{{}^{r}C_2 \ {}^{r+1}C_2}$, $a = \lim_{n \to \infty} f(n)$ and $x^2 - \left(2a - \frac{1}{2}\right)x + t = 0$ has two positive roots α and β.

14. If value of $f(7) + f(8)$ is $\frac{p}{q}$ where p and q are relatively prime, then $(p - q)$ is :

(a) 53 (b) 55 (c) 57 (d) 59

15. Minimum value of $\frac{4}{\alpha} + \frac{1}{\beta}$ is :

(a) 2 (b) 6 (c) 3 (d) 4

Paragraph for Question Nos. 16 to 17

Given the sequence of number $a_1, a_2, a_3, \ldots\ldots, a_{1005}$

which satisfy $\frac{a_1}{a_1+1} = \frac{a_2}{a_2+3} = \frac{a_3}{a_3+5} = \ldots\ldots = \frac{a_{1005}}{a_{1005}+2009}$

Also $a_1 + a_2 + a_3 + \ldots\ldots + a_{1005} = 2010$

16. Nature of the sequence is :

(a) A.P. (b) G.P. (c) A.G.P. (d) H.P.

17. 21st term of the sequence is equal to :

(a) $\frac{86}{1005}$ (b) $\frac{83}{1005}$ (c) $\frac{82}{1005}$ (d) $\frac{79}{1005}$

Answers

1. (b)	**2.** (a)	**3.** (b)	**4.** (d)	**5.** (c)	**6.** (a)	**7.** (a)	**8.** (b)	**9.** (c)	**10.** (c)
11. (c)	**12.** (b)	**13.** (b)	**14.** (d)	**15.** (b)	**16.** (a)	**17.** (c)			

Exercise-4 : Matching Type Problems

1.

	Column-I		Column-II
(A)	If three unequal numbers a, b, c are in A.P. and $b-a$, $c-b$, a are in G.P., then $\frac{a^3+b^3+c^3}{3abc}$ is equal to	**(P)**	1
(B)	Let x be the arithmetic mean and y, z be two geometric means between any two positive numbers, then $\frac{y^3+z^3}{2xyz}$ is equal to	**(Q)**	4
(C)	If a, b, c be three positive number which form three successive terms of a G.P. and $c > 4b-3a$, then the common ratio of the G.P. can be equal to	**(R)**	2
(D)	Number of integral values of x satisfying inequality, $-7x^2+8x-9>0$ is	**(S)**	0

2.

	Column-I		Column-II
(A)	The sequence a, b, 10, c, d are in A.P., then $a+b+c+d=$	**(P)**	6
(B)	Six G.M.'s are inserted between 2 and 5, if their product can be expressed as $(10)^n$. Then $n=$	**(Q)**	2
(C)	Let $a_1, a_2, a_3, \ldots, a_{10}$ are in A.P. and $h_1, h_2, h_3, \ldots, h_{10}$ are in H.P. such that $a_1=h_1=1$ and $a_{10}=h_{10}=6$, then $a_4h_7=$	**(R)**	3
(D)	If $\log_3 2$, $\log_3(2^x-5)$ and $\log_3\left(2^x-\frac{7}{2}\right)$ are in A.P., then $x=$	**(S)**	20
		(T)	40

3.

	Column-I		Column-II
(A)	The number of real values of x such that three numbers 2^x, 2^{x^2} and 2^{x^3} form a non-constant arithmetic progression in that order, is	**(P)**	0
(B)	Let $S=(a_2-a_3)\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right)$ where $a_1, a_2, a_3, \ldots, a_n$ are n consecutive terms of an A.P. and $a_i>0\ \forall i\in\{1, 2, \ldots, n\}$. If $a_1=225$, $a_n=400$, then the value of $S+7$ is equal to	**(Q)**	1

(C)	Let S_n denote the sum of first n terms of an non constant A.P. and $S_{2n} = 3S_n$, then $\frac{S_{3n}}{2S_n}$ is equal to	(R)	2
(D)	If t_1, t_2, t_3, t_4 and t_5 are first 5 terms of an A.P., then $\frac{4(t_1 - t_2 - t_4) + 6t_3 + t_5}{3t_1}$ is equal to	(S)	3
		(T)	4

4. Column-I contains S and **Column-II** gives last digit of S.

	Column-I		Column-II
(A)	$S = \sum_{n=1}^{11} (2n-1)^2$	(P)	0
(B)	$S = \sum_{n=1}^{10} (2n-1)^3$	(Q)	1
(C)	$S = \sum_{n=1}^{18} (2n-1)^2(-1)^n$	(R)	3
(D)	$S = \sum_{n=1}^{15} (2n-1)^3(-1)^{n-1}$	(S)	5
		(T)	8

5.

	Column-I		Column-II
(A)	If $x, y \in R^+$ satisfy $\log_8 x + \log_4 y^2 = 5$ and $\log_8 y + \log_4 x^2 = 7$ then the value of $\frac{x^2 + y^2}{2080} =$	(P)	6
(B)	In ΔABC A, B, C are in A.P. and sides a, b and c are in G.P. then $a^2(b-c) + b^2(c-a) + c^2(a-b) =$	(Q)	3
(C)	If a, b, c are three positive real numbers then the minimum value of $\frac{b+c}{a} + \frac{a+c}{b} + \frac{a+b}{c}$ is	(R)	0
(D)	In ΔABC, $(a+b+c)(b+c-a) = \lambda bc$ where $\lambda \in I$, then greatest value of λ is	(S)	2

6. Let $f(n)=1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+......+\frac{1}{n}$ such that $P(n)f(n+2)=P(n)f(n)+q(n)$. Where $P(n)$, $Q(n)$ are polynomials of least possible degree and $P(n)$ has leading coefficient unity. Then match the following Column-I with Column-II.

	Column-I		Column-II
(A)	$\sum_{n=1}^{m}\frac{p(n)-2}{n}$	**(P)**	$\frac{m(m+1)}{2}$
(B)	$\sum_{n=1}^{m}\frac{q(n)-3}{2}$	**(Q)**	$\frac{5m(m+7)}{2}$
(C)	$\sum_{n=1}^{m}\frac{p(n)+q^2(n)-11}{n}$	**(R)**	$\frac{3m(m+7)}{2}$
(D)	$\sum_{n=1}^{m}\frac{q^2(n)-p(n)-7}{n}$	**(S)**	$\frac{m(m+7)}{2}$

Answers

1.	A→R; B→P; C→Q; D→S
2.	A→R, B→R, C→P, D→R
3.	A→P, B→R, C→S, D→Q
4.	A→Q; B→P; C→T; D→S
5.	A→S; B→R; C→P; D→Q
6.	A→S; B→P; C→Q; D→R

Exercise-5 : Subjective Type Problems

1. Let a, b, c, d are four distinct consecutive numbers in A.P. The complete set of values of x for which $2(a-b)+x(b-c)^2+(c-a)^3 = 2(a-d)+(b-d)^2+(c-d)^3$ is true is $(-\infty, \alpha] \cup [\beta, \infty)$, then $|\alpha|$ is equal to :

2. The sum of all digits of n for which $\sum_{r=1}^{n} r\,2^r = 2+2^{n+10}$ is :

3. If $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{r+2}{2^{r+1} r(r+1)} = \frac{1}{k}$, then $k =$

4. The value of $\sum_{r=1}^{\infty} \frac{8r}{4r^4+1}$ is equal to :

5. Three distinct non-zero real numbers form an A.P. and the squares of these numbers taken in same order form a G.P. If possible common ratio of G.P. are $3 \pm \sqrt{n}$, $n \in N$ then $n =$

6. If $\sqrt{\underbrace{(1111......1)}_{2n \text{ times}} - \underbrace{(222......2)}_{n \text{ times}}} = \underbrace{PPP......P}_{n \text{ times}}$ then $P =$

7. In an increasing sequence of four positive integers, the first 3 terms are in A.P., the last 3 terms are in G.P. and the fourth term exceed the first term by 30, then the common difference of A.P. lying in interval [1, 9] is :

8. The limit of $\frac{1}{n^4}\sum_{k=1}^{n} k(k+2)(k+4)$ as $n \to \infty$ is equal to $\frac{1}{\lambda}$, then $\lambda =$

9. What is the last digit of $1+2+3+......+n$ if the last digit of $1^3+2^3+......+n^3$ is 1?

10. Three distinct positive numbers a, b, c are in G.P., while $\log_c a$, $\log_b c$, $\log_a b$ are in A.P. with non-zero common difference d, then $2d =$

11. The numbers $\frac{1}{3}, \frac{1}{3}\log_x y, \frac{1}{3}\log_y z, \frac{1}{7}\log_z x$ are in H.P. If $y = x^r$ and $z = x^s$, then $4(r+s) =$

12. If $\sum_{k=1}^{\infty} \frac{k^2}{3^k} = \frac{p}{q}$; where p and q are relatively prime positive integers. Find the value of $(p+q)$.

13. The sum of the terms of an infinitely decreasing Geometric Progression (GP) is equal to the greatest value of the function $f(x) = x^3+3x-9$ when $x \in [-4, 3]$ and the difference between the first and second term is $f'(0)$. The common ratio $r = \frac{p}{q}$ where p and q are relatively prime positive integers. Find $(p+q)$.

14. A cricketer has to score 4500 runs. Let a_n denotes the number of runs he scores in the n^{th} match. If $a_1 = a_2 = ... a_{10} = 150$ and $a_{10}, a_{11}, a_{12}....$ are in A.P. with common difference (-2). If N be the total number of matches played by him to score 4500 runs. Find the sum of the digits of N.

15. If $x = 10\sum_{n=3}^{100} \frac{1}{n^2-4}$, then $[x]$ = (where $[\cdot]$ denotes greatest integer function)

16. Let $f(n) = \frac{4n+\sqrt{4n^2-1}}{\sqrt{2n+1}+\sqrt{2n-1}}, n \in N$ then the remainder when $f(1)+f(2)+f(3)+\ldots\ldots+f(60)$ is divided by 9 is.

17. Find the sum of series $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{9}+\frac{1}{12}+\ldots\ldots\infty$, where the terms are the reciprocals of the positive integers whose only prime factors are two's and three's :

18. Let $a_1, a_2, a_3, \ldots\ldots\ldots, a_n$ be real numbers in arithmatic progression such that $a_1 = 15$ and a_2 is an integer. Given $\sum_{r=1}^{10}(a_r)^2 = 1185$. If $S_n = \sum_{r=1}^{n} a_r$ and maximum value of n is N for which $S_n \geq S_{(n-1)}$, then find $N-10$.

19. Let the roots of the equation $24x^3 - 14x^2 + kx + 3 = 0$ form a geometric sequence of real numbers. If absolute value of k lies between the roots of the equation $x^2 + \alpha^2 x - 112 = 0$, then the largest integral value of α is :

20. How many ordered pair(s) satisfy $\log\left(x^3+\frac{1}{3}y^3+\frac{1}{9}\right) = \log x + \log y$

21. Let a and b be positive integers. The value of xyz is 55 and $\frac{343}{55}$ when a, x, y, z, b are in arithmatic and harmonic progression respectively. Find the value of $(a+b)$

Answers

1.	8	2.	9	3.	2	4.	2	5.	8	6.	3	7.	9
8.	4	9.	1	10.	3	11.	6	12.	5	13.	5	14.	7
15.	5	16.	8	17.	3	18.	6	19.	2	20.	1	21.	8

❑❑❑

Chapter 10 till end (Chapter 26) is in Part 2